HW 6

(Q-1) Prove $\kappa^2 = \kappa_n^2 + \kappa_g^2$.

(Q-2) Show that the matrix (L_{ij}) for the surface of revolution from (HW-4, Q-2) is

$$\frac{1}{\sqrt{\dot{r}^2 + \dot{z}^2}} \begin{pmatrix} \dot{r}\ddot{z} - \ddot{r}\dot{z} & 0 \\ 0 & r\dot{z} \end{pmatrix}.$$

(Q-3) Prove that for a surface of revolution $\det(L_{ij}) \equiv 0$ if and only if each meridian is a straight line.

(Q-4) Let $\overline{L}_{\alpha\beta}$ be the corresponding expression for L_{ij} in a coordinate system V, and let $f \colon U \to V$ be a coordinate transformation. Prove

$$L_{ij} = \pm \sum \overline{L}_{\alpha\beta} \frac{\partial v^\alpha}{\partial u^i} \frac{\partial v^\beta}{\partial u^j}.$$

(Q-5) For the patch $x(u, v) = (u, v, u^2 + v^2)$ find the normal curvature of the curve $\gamma(t) = x(t^2, t)$ at $t = 1$.

(Q-6) Consider the plane as the simple surface $x(r, s) = (r, s, 0)$. Prove that the geodesic curvature of a curve in the plane is its plane curvature.

(Q-7) The sphere is a surface of revolution. Find the geodesic curvature of a circle of latitude.

(Q-8) Let γ be a curve on a sphere. Prove that κ_n is constant.

(Q-9) Let γ be a curve on a sphere with κ_g constant. Prove γ is a circle.

(Q-10) Does the sign of the geodesic curvature have any meaning in a coordinate patch? In a surface?