HW 5

(Q-1) The coordinate patch from (HW-4, Q-2) does not cover the entire surface of revolution. It omits points that would correspond to $\theta = \pm\pi$. Define a second coordinate patch with $0 < \phi < 2\pi$, check the overlap condition, and thus show that the surface of revolution is a surface.

(Q-2) (HW-4, Q-1) gives a coordinate patch for a torus, which is a the surface of a donut. Find three more patches making the entire torus a surface.

(Q-3) Note that every point in S^2 is either in the image of the patch from (HW-4, Q-8) or the patch $x(u^1, u^2) = (u^1, u^2, \sqrt{1-(u^1)^2-(u^2)^2}$ with $(u^1)^2 + (u^2)^2 < 1$. Show that these two patches make S^2 into a surface by computing the coordinate transformation on the overlap.

(Q-4) Describe some possible parametrizations of the ellipsoid

$$x^2/a^2 + y^2/b^2 + z^2/c^2 = 1.$$

(Q-5) Give another coordinate patch for the surface from (HW-4, Q-7) so that the points $\theta = \pm\pi$ are included. Note that (HW-4, Q-7) implies that it is impossible to choose a unit normal on this entire surface in a continuous fashion.

(Q-6) Prove that the metric coefficients for the surface of revolution from (HW-4, Q-2) are given by

$$\begin{pmatrix} \dot{r}^2 + \dot{z}^2 & 0 \\ 0 & r^2 \end{pmatrix}$$

(Q-7) Compute the metric coefficients of the parametrization of S^2 from (HW-4, Q-8).

(Q-8) Compute the inverse of the metric coefficients from **??** and **??**.

(Q-9) A coordinatge patch is called an orthogonal net if u^1-curves meet the u^2-curves at right angles. Prove the meridians and longitudes of a surface of revolution form an orthogonal net.

(Q-10) Let x, y be the Cartesian coordinates and r, θ be the polar coordinates on the plane $\mathbb{R}^2$. Prove $x = r\cos\theta, y = r\sin\theta$ is a coordinate transformation. Show the metric coefficients of the plane with respect to Cartesian coordinates is

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and compute the metric coefficients of the plane for polar coordinates.