HW 4

(Q-1) Define

$$x(u^1,u^2) = \big((2+\cos u^1)\cos u^2, (2+\cos u^1)\sin u^2, \sin u^1\big)$$

with $-\pi < u^1, u^2 < \pi$. Prove x is a simple surface. Compute x_1, x_2, n as functions of u^1, u^2.

(Q-2) Let $\alpha(t) = (r(t), z(t))$ be a regular injective curve, and consider

$$x(t,\theta) = (r(t)\cos\theta, r(t)\sin\theta, z(t)),$$

with $-\pi < \theta < \pi$. Prove it is a simple surface. Compute x_1, x_2, n.

(Q-3) The surface from (Q-2) is called a surface of revolution, and the t-curves are called meridians, and the θ-curves are called circles of latitudes, and the z-axis the axis of revolution. Prove the surface from (Q-1) is a surface of revolution, and compute its meridians and latitudes.

(Q-4) Let $U = \{(u^1,u^2) \mid 0 < u^1, 0 < u^2 < 2\pi\}$ and

$$x(u^1,u^2) = (u^1\cos u^2, u^1\sin u^2, u^1 + u^2).$$

Prove x is a simple surface.

(Q-5) Let $\theta(u^1)$ be a C^k function. Prove

$$x(u^1,u^2) = (u^2\cos\theta(u^1), u^2\sin\theta(u^1), u^1)$$

is a simple surface.

(Q-6) Let $x(u^1,u^2) = (u^1+u^2, u^1-u^2, u^1u^2)$. Show x is a simple surface. Find the normal and the equation of the tangent plane art $u^1 = 1, u^2 = 2$.

(Q-7) Let

$$x(\theta, v) = (\cos\theta, \sin\theta, 0) + v(\sin\tfrac{\theta}{2}\cos\theta, \sin\tfrac{\theta}{2}\sin\theta, \cos\tfrac{\theta}{2})$$

with $\theta \in (-\pi,\pi), v \in (-\frac{1}{2}, \frac{1}{2})$. Compute $n(\theta,0)$ and show that

$$\lim_{\theta\to-\pi} n(\theta,0) = -\lim_{\theta\to\pi} n(\theta,0)$$

while

$$\lim_{\theta\to-\pi} x(\theta,0) = \lim_{\theta\to\pi} x(\theta,0).$$

Draw the surface.

(Q-8) Let $S^2 = \{(u,v,w) \in \mathbb{R}^3 \mid u^2+v^2+w^2 = 1\}$ and $\mathbb{R}^2 = \{(u,v,w) \in \mathbb{R}^3 \mid w = 0\}$. The line joining $(u,v,0)$ and $(0,0,1)$ intersects S^2 is a point other than $(0,0,1)$; call this point $x(u,v)$ and prove $x\colon \mathbb{R}^2 \to \mathbb{R}^3$ is a simple surface. (The inverse mapping is called the stereographic projection.)

(Q-9) Let

$$x(\theta,\phi) = (\sin\phi\cos\theta, 2\sin\phi\sin\theta, 3\cos\phi), \qquad \theta \in (-\pi,\pi), \phi \in (-\pi/2, \pi/2).$$

Show x is a simple surface. What is it?

(Q-10) Let

$$x(u,v) = (\sqrt{1-u^2}\cos v, \sqrt{1-u^2}\sin v, u), \qquad u \in (-1,1), v \in (-\pi,\pi).$$

Show x is a simple surface. What is it?