HW 3

(Q-1) Compute the Frenet-Serret apparatus for the non-unit speed curve $\beta(t) = (t, t^2, t^3)$.

(Q-2) Find the curvature and torsion for the non-unit speed curve $(e^t \cos t, e^t \sin t, e^t)$.

(Q-3) Compute $\int_C (xy+1)dx + (x^2/y)dy$ along each of the following curves:
- C is given by $\alpha(t) = (t, t)$, $0 \le t \le 1$.
- C is given by $\beta(t) = (t^2, t^3)$, $0 \le t \le 1$.

(Q-4) Compute $\int_C (xdy - ydx)/(x^2+y^2)$ where C is the straight line segment:
- from $(0,1)$ to $(1,0)$;
- from $(1,0)$ to $(1,1)$;
- from $(1,1)$ to $(0,1)$;

(Q-5) Verify Green's theorem for $\int_C (xdy - ydx)/(x^2+y^2)$ where C is the triangle with vertices $(0,1), (1,0), (1,1)$.

(Q-6) For a unit speed plane curve α, prove that $\kappa(s) = |k(s)|$, and if $\kappa(s) \neq 0$, then $n(s) = \pm N(s)$, and $n'(s) = -k(s)T(s)$.

(Q-7) If $\beta(t)$ is a closed curve with period a, and $\alpha(s)$ is its arc-length reparametrization, then prove that $\alpha(s)$ is closed with period $L = \int_0^a |d\beta/dt| dt$.

(Q-8) If α is a unit speed closed plane curve with period L, prove that there is a well-defined function $\theta\colon [0,L] \to \mathbb{R}$, such that $T(s) = (\cos\theta(s), \sin\theta(s))$.

(Q-9) If α is a simple closed unit-speed plane curve, prove that tangent direction $T\colon [0,L] \to S^1$ is an onto map.

(Q-10) Give an example of a non-simple closed unit-speed plane curve where the tangent direction $T\colon [0,L] \to S^1$ is not an onto map.