HW 2

(Q-1) Show that $\alpha(s) = \frac{1}{13}\big(5\cos s, 8-13\sin s, -12\cos s\big)$ is a unit speed curve and compute its Frenet-Serret apparatus.

(Q-2) Show that $\alpha(s) = \big(\frac{(1+s)^{3/2}}{3}, \frac{(1-s)^{3/2}}{3}, \frac{s}{\sqrt{2}}\big)$ is a unit speed curve and compute its Frenet-Serret apparatus.

(Q-3) Let $\alpha(s)$ be a curve in the (x,y)-plane. Prove that if $\kappa \neq 0$ then $\tau = 0$. (Hint: First prove $B = \pm(0,0,1)$.)

(Q-4) $\alpha(s) = (x(s), y(s), 0)$ be a unit speed curve. Prove $\kappa = |x'y'' - x''y'|$.

(Q-5) Let $\alpha(s)$ be a unit speed curve with $\kappa \neq 0$. Prove

$$\tau = \frac{[\alpha', \alpha'', \alpha''']}{\langle \alpha'', \alpha'' \rangle}.$$

(Q-6) Find the equation of the normal plane to $\alpha(t) = (e^t, \cos t, 3t^2)$ at $t = 1$. (Note: t is not the arc length.)

(Q-7) Let $\alpha(s)$ be a unit speed curve with $\kappa \neq 0$. Find a vector $w(s)$ such that $T' = w \times T$, $N' = w \times N$, $B' = w \times B$.

(Q-8) Prove that $\alpha(s)$ is a straight line if and only if all its tangent lines are parallel.

(Q-9) Let

$$f(t) = \begin{cases} e^{-1/t^2} & \text{if } t \neq 0 \\ 0 & \text{if } t = 0. \end{cases}$$

You may assume f is C^∞ with all higher derivatives 0 at 0. Let $\alpha(t)$ be given by

$$\alpha(t) = \begin{cases} (t, f(t), 0) & \text{if } t \leq 0 \\ (t, 0, f(t)) & \text{if } t > 0. \end{cases}$$

Prove α is regular and C^∞. Show $\kappa = 0$ at $t = 0$. (Note: t is not the arc length.)

(Q-10) Find a unit speed curve $\alpha(s)$ with $\kappa(s) = 1/(1+s^2)$ and $\tau = 0$.