HW 1

(Q-1) Let $\alpha(\theta) = (e^\theta \cos\theta, e^\theta \sin\theta, 0)$. Prove that the angle between α and T is constant.

(Q-2) Let $\alpha(t)$ be a regular curve. Suppose there is a point $a \in \mathbb{R}^3$ such that $\alpha(t) - a$ is orthogonal to $T(t)$ for all t. Prove that $\alpha(t)$ lies on a sphere. (Hint: what is the center of the sphere?)

(Q-3) Consider the function $\alpha\colon \mathbb{R} \to \mathbb{R}^3$ by $\alpha(t) = (t^2, t^3, 0)$. Prove α is C^1 but not regular. Show that the image of α has a sharp corner by graphing α.

(Q-4) Let $g\colon (0, \infty) \to (0, 1)$ be given by $g(r) = r^2/(r^2 + 1)$. Is this a reparametrization?

(Q-5) Find the arc length of the circular helix $\alpha(t) = (r \cos t, r \sin t, ht)$, $0 \le t \le 10$.

(Q-6) Reparametrize the circular helix $\alpha(t) = (r \cos t, r \sin t, ht)$ by arc length.

(Q-7) Find the arc length of $\alpha(t) = (2 \cosh 3t, -2 \sinh 3t, 6t)$, $0 \le t \le 5$.

(Q-8) Reparametrize the curve $\alpha(t) = (e^t \cos t, e^t \sin t, e^t)$ by arc length.

(Q-9) Prove that the following is a unit speed curve.

$$\alpha(s) = \tfrac{1}{2}\big(s + \sqrt{s^2+1}, (s + \sqrt{s^2+1})^{-1}, \sqrt{2}\ln(s + \sqrt{s^2+1})\big).$$

(Q-10) Let $\alpha(t)$ be a regular curve with $|d\alpha/dt| = a$, where a is a fixed positive constant. If s is the arc length measured from some point, prove $t = c + s/a$ for some constant c.