(Q-1) Let the triangle $ABC$ be inscribed in a circle, let $P$ denote the centroid of the triangle, and let $O$ denote the circumcenter. Suppose that $A, B, C$ have coordinates $(0, 0)$, $(a, 0)$, and $(b, c)$ respectively.

(a) Express the coordinates of $P$ and $O$ in terms of $a, b, c$.

(b) Extend the line segments $AP$, $BP$, and $CP$ to the circle in points $D$, $E$, and $F$, respectively. Show that

$$\frac{AP}{PD} + \frac{BP}{PE} + \frac{CP}{PF} = 3.$$

(Hint: One way to proceed is the following. Let $x$ denote $OP$, and let $R$ be the radius of the circumcircle. Then first show that

$$\frac{AP}{PD} + \frac{BP}{PE} + \frac{CP}{PF} = \frac{AP^2 + BP^2 + CP^2}{R^2 - x^2}.$$

Then express each of the terms on the right hand side in terms of $a, b, c$ (using results from the previous part).)

(Q-2) Find the relation that must hold between the parameters $a, b, c$ so that the line $x/a + y/b = 1$ will be tangent to the circle $x^2 + y^2 = c^2$.

(Q-3) A parabola with equation $y^2 = ax$ is cut in four points by the circle $(x-h)^2+(y-k)^2 = r^2$. Determine the product of the distances of the four points of intersection from the axis of the parabola.

(Q-4) The sides $AD, AB, CB, CD$ of the quadrilateral $ABCD$ are divided by points $E, F, G, H$ so that $AE : ED = AF : FB = CG : GB = CH : HD$. Prove that $EFGH$ is a parallelogram.

(Q-5) On the sides of an arbitrary parallelogram, squares are constructed lying exterior to it. Prove that their centers $M_1, M_2, M_3, M_4$ are themselves vertices of a square.

(Q-6) On the sides of an arbitrary quadrilateral $ABCD$, equilateral triangles $ABM_1$, $BCM_2$, $CDM_3$, $DAM_4$ are constructed so that the first and third are exterior on the quadrilateral, while the second and fourth are on the same side of $BC$ and $DA$ as the quadrilateral itself. Prove that the quadrilateral $M_1M_2M_3M_4$ is a parallelogram.

(Q-7) In a tetrahedron, two pairs of opposite edges are orthogonal. Prove that the third pair of opposite edges must also be orthogonal.

(Q-8) Given a point $P$ on the circumference of a unit circle and the vertices $A_1, A_2, \ldots, A_n$ of a regular polygon of $n$ sides inscribed in the unit circle, prove that $PA_1^4 + PA_2^4 + \cdots + PA_n^4$ is constant (i.e., independent of the position of $P$ on the circumference).

(Q-9) Let $G$ be the centroid of a triangle $ABC$. Prove that

$$3(GA^2 + GB^2 + GC^2) = AB^2 + BC^2 + CA^2.$$

(Q-10) Let $ABCDEF$ be a hexagon in a circle of radius $r$. Show that if $AB = CD = EF = r$, then the midpoints of $BC$, $DE$, and $FA$ are the vertices of an equilateral triangle.