(Q-1) Find the power series expansion for $1/(x^2+5x+6)$.

(Q-2) Solve the recurrence relation $a_0 = 1, a_1 = 0, a_2 = -5$, and for $n \geq 3$

$$a_n = 4a_{n-1} - 5a_{n-2} + 2a_{n-3}.$$

(Q-3) Sum the finite series $a_0 + a_1 + \cdots + a_n$, where $a_0 = 2, a_1 = 17$, and for $i > 1$, $a_i = 7a_{i-1} - 12a_{i-2}$.

(Q-4) Solve the recurrence relation:

$$a_n = (\sqrt{a_{n-1}} + 2\sqrt{a_{n-2}})^2$$

with initial condition $a_0 = a_1 = 1$.

(Q-5) Solve the recurrence relation:

$$a_n = \sqrt{\frac{a_{n-2}}{a_{n-1}}}$$

with initial condition $a_0 = 8, a_1 = 1/(2\sqrt{2})$. (Hint: Let $b_n = \ln a_n$.)

(Q-6) Find the general term formula for the sequence $(y_n)_{n\geq 0}$ with $y_0 = 1$ and $y_n = ay_{n-1} + b^n$ for $n \geq 1$, where a and b are two fixed distinct real numbers.

(Q-7) Prove the inclusion-exclusion principle:

$$|A_1 \cup \cdots \cup A_n| = \sum_i |A_i| - \sum_{i<j} |A_i \cap A_j| + \sum_{i<j<k} |A_i \cap A_j \cap A_k| - \cdots + (-1)^{n-1}|A_1 \cap \cdots \cap A_n|$$

by induction on n.

(Q-8) How many positive integers less than 1000 are not divisible by 2, 3, or 7?

(Q-9) What is the probabiliy that the sum of two (uniform) randomly chosen numbers in the interval $[0, 1]$ does not exceed 1 and their product does not exceed $\frac{2}{9}$?

(Q-10) Let $\alpha \in (0, 1)$. If two points are selected (uniformly) at random from a straight line segement of length 1, what is the probability that the distance between them is at least α?