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- Exercises are taken from J. Rogawski, C. Adams, R. Franzosa *Calculus, Multivariable*, 4th Ed., W. H. Freeman & Company.

17.4 Parametrized Surfaces and Surface Integrals

In Exercises 1–4, calculate $\iint_S f(x,y,z)\,\mathrm{d}S$ *for the given surface and function.*

1. $G(u,v) = (u\cos v, u\sin v, 2u)$, $\quad 0 \le u \le 1$, $\quad 0 \le u \le 1$; $f(x,y,z) = z(x^2+y^2)$.

2. $y = 4 - z^2$, $\quad 0 \le x \le z \le 3$; $\quad f(x,y,z) = -2 - 8z$.

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3. $x^2 + y^2 + z^2 = 4, \quad z \geq 0; \qquad f(x,y,z) = x^2.$

Hint: You might find $\int \sin^3 \phi \, d\phi = \frac{1}{12} \cos 3\phi - \frac{3}{4} \cos \phi + C$ *useful.*

4. Part of the plane $x + y + 2z = 4$, where $x, y, z \geq 0$; $f(x,y,z) = x + y + 3z$.

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5. (Exercise 17.4-30) Use spherical coordinates to compute the surface area of a sphere of radius R.

6. (Exercise 17.4-28) Let $\mathcal{S}$ be the sphere of radius R centered at the origin. Explain using symmetry:

$$\iint_{\mathcal{S}} x^2 \, dS = \iint_{\mathcal{S}} y^2 \, dS = \iint_{\mathcal{S}} z^2 \, dS$$

Then show that

$$\iint_{\mathcal{S}} x^2 \, dS = \frac{4}{3}\pi R^4$$

by adding the integrals.

7. Let $\mathcal{S}$ be the portion of the sphere $x^2 + y^2 + z^2 = 16$, where $4 \leq x^2 + y^2 \leq 9$ and $z \geq 0$. Compute

$$\iint_{\mathcal{S}} \frac{1}{z} \, dS.$$

17.5 Surface Integrals of Vector Fields

1. (Exercise 17.5-1) Let $\mathbf{F} = \langle z, 0, y \rangle$, and $\mathcal{S}$ be the oriented surface parametrized by $G(u,v) = (u^2 - v, u, v^2)$ for $0 \leq u \leq 2$, $-1 \leq v \leq 4$. Calculate:

(a) $\mathbf{N}$ and $\mathbf{F} \cdot \mathbf{N}$ as functions of u and v

(b) The normal component of $\mathbf{F}$ to the surface at $P = (3, 2, 1) = G(2, 1)$

(c) $\iint_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S}$.

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In Exercises 2–5, calculate $\iint_S \mathbf{F} \cdot d\mathbf{S}$ *for the given oriented surface.*

2. $\mathbf{F} = \langle z, x, y \rangle$, plane $2x - 5y + z = 1$, $0 \le x \le 1$, $0 \le y \le 1$, upward-pointing normal.

3. $\mathbf{F} = \langle e^z, y, z \rangle$, $G(r,s) = (rs, r+s, r)$, $0 \le r \le 1$, $0 \le s \le 1$, positively oriented.

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4. $\mathbf{F} = \langle z, 2, 0 \rangle$, part of the sphere $x^2 + y^2 + z^2 = 4$, where $x \geq 0$, $y \geq 0$, $z \geq 0$, outward-pointing normal.

5. $\mathbf{F} = \langle y, z, 0 \rangle$, $G(u, v) = (u^3 - v, u + v, v^2)$, $0 \leq u \leq 2$, $0 \leq v \leq 3$, downward-pointing normal.

6. Let $\mathcal{S}$ be the portion of the ellipsoid

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{4}\right)^2 + \left(\frac{z}{3}\right)^2 = 1$$

where $x, y, z \geq 0$. Calculate the flux of $\mathbf{F} = x\mathbf{i}$ over $\mathcal{S}$.

Hint: Parametrize $\mathcal{S}$ using a modified form of spherical coordinates.

7. Recall that

$$\mathbf{e}_r = \frac{1}{r}\langle x, y, z\rangle, \qquad r = \sqrt{x^2 + y^2 + z^2}$$

is the unit radial vector field. Calculate the flux of $\mathbf{F} = r^{-2}\mathbf{e}_r$ through a sphere of radius R centered at the origin.

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18.1 Green's Theorem

In Exercises 1–5, use Green's Theorem to evaluate the line integral. Orient the curve counterclockwise unless otherwise indicated.

1. $\oint_{\mathcal{C}} y^2\,\mathrm{d}x + x^2\,\mathrm{d}y$, where $\mathcal{C}$ is the boundary of the unit square $[0,1] \times [0,1]$.

2. $\oint_{\mathcal{C}} 7y\,\mathrm{d}x - 3x\,\mathrm{d}y$, where $\mathcal{C}$ is the boundary of the triangle with vertices $(-1,0)$, $(1,0)$, and $(0,1)$.

3. $\oint_{\mathcal{C}} e^{x+y}\,\mathrm{d}x + e^{-y^2}\,\mathrm{d}y$, where $\mathcal{C}$ is the boundary of the triangle $(0,0)$, $(1,0)$, and $(1,1)$.

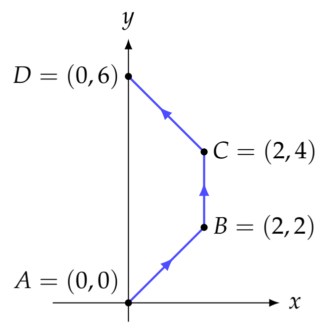
4. $\oint_{\mathcal{C}} x^2 y\,\mathrm{d}x$, where $\mathcal{C}$ is the unit circle centered at the origin.

5. $\oint_{\mathcal{C}} \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y) = \langle x+y, x^2-y\rangle$ and $\mathcal{C}$ is the boundary of the region enclosed by $y = x^2$ and $y = \sqrt{x}$ for $0 \le x \le 1$.

6. Evaluate

$$I = \int_{\mathcal{C}} (5\sin x + 2y)\, dx + (4x+y)\, dy$$

for the non-closed path $\mathcal{C}_1$ joining from A to B to C to D:



Hint: Let $\mathcal{C}_2$ be the line segment from A to D. Then $\mathcal{C}_1 - \mathcal{C}_2$ is a closed path oriented counterclockwise. Apply the Green's Theorem to this closed path, and use this to compute I.

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7. Use $\oint_C y\,\mathrm{d}x$ to compute the area of the ellipse

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1.$$

8. Use $\frac{1}{2}\oint_C x\,\mathrm{d}y - y\,\mathrm{d}x$ to compute the area of the region bounded by the curve $r = \sin 2\theta$ in polar coordinates: