Practice Midterm 2

Name: ______________________________

Student ID: __ __ __ __ __ __ __ __ __

Section:

Tuesday	Thursday
☐ 2A	☐ 2B
☐ 2C	☐ 2D
☐ 2E	☐ 2F

Instructions:

- Do not open this exam until instructed to do so.
- You have 50 minutes to complete the exam.
- Please print your name and student ID number above and check the box of your discussion section.
- **You may not use calculators**, books, notes, or any other material to help you. Please make sure your **phone is silenced and stowed** where you cannot see it.
- We will only grade your work within the pages that are originally included.
- In each of the questions 2 through 5, you must **show your work** to receive full credit. Please write your solutions in the space below the questions. You must indicate if you go over the pages.

Please do not write below this line.

Question	Points	Score
1	8	
2	7	
3	8	
4	5	
5	7	
6	8	
Total	43	

1. (8 pts) Consider the curve $\mathcal{C}$ parametrized by

$$\mathbf{r}(t) = \langle 3t\cos t,\ 3t\sin t,\ (2t)^{3/2}\rangle, \qquad 0 \le t \le 1.$$

(a) (4 pts) Compute the length of $\mathcal{C}$.

(b) (4 pts) Evaluate $\displaystyle\int_{\mathcal{C}} z^2\, \mathrm{d}s$.

2. (7 pts) Let $\mathcal{C}$ be the oriented curve which lies in the first quadrant of the xy-plane, is described by $x^2 + 4y^2 = 4$ and $z = 0$, starts at $(2, 0, 0)$, and ends at $(0, 1, 0)$. Let

$$\mathbf{F}(x, y, z) = \langle 0,\ 3xy,\ 0 \rangle.$$

Evaluate $\displaystyle\int_{\mathcal{C}} \mathbf{F} \cdot \mathrm{d}\mathbf{r}$.

3. (8 pts) Let $\mathcal{C}$ be the oriented curve parametrized by

$$\mathbf{r}(t) = \langle t^{42},\ \cos(\pi t^{2020}),\ e^{t(1-t)} \rangle, \qquad 0 \le t \le 1,$$

so that $\mathcal{C}$ starts at $\mathbf{r}(0) = \langle 0, 1, 1 \rangle$ and ends at $\mathbf{r}(1) = \langle 1, -1, 1 \rangle$. Evaluate $\displaystyle\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$, where

$$\mathbf{F}(x, y, z) = \left\langle 2xyz + e^z,\ x^2 z,\ x^2 y + xe^z \right\rangle.$$

4. (5 pts) Let $\mathbf{F}(x,y) = \langle F_1(x,y), F_2(x,y)\rangle$ be a vector field which is defined and continuously differentiable on the xy-plane except at the points $(-2,0)$ and $(2,0)$. Suppose that we know that, at every point in the domain of $\mathbf{F}$,

$$\frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}$$

holds true. Let $\mathcal{C}$ be the boundary of the rectangle $[-1,1]\times[1,2]$ oriented clockwise. Compute

$$\oint_{\mathcal{C}} \mathbf{F}\cdot d\mathbf{r}$$

and **justify** your answer. (The point will be scarce without any justification.)

5. (7 pts) Let $\mathcal{S}$ be part of the plane $2x + 2y + z = 6$, where $x, y, z \geq 0$. Evaluate

$$\iint_{\mathcal{S}} (x + y + z)\,\mathrm{d}S$$

6. (8 pts) Let $\mathcal{S}$ be the portion of the surface $4x^2 + y^2 = 4$, where $0 \leq z \leq 4$, with outward-pointing normal. Compute $\iint_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S}$, where

$$\mathbf{F}(x, y, z) = \langle 2x,\ 3y,\ e^z \rangle.$$

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