Midterm 1 (Practice)

Name: ______________________________

Student ID: __ __ __ __ __ __ __ __ __

Section:

Tuesday	Thursday
☐ 2A	☐ 2B
☐ 2C	☐ 2D
☐ 2E	☐ 2F

Instructions:

- Do not open this exam until instructed to do so.
- You have 50 minutes to complete the exam.
- Please print your name and student ID number above and check the box of your discussion section.
- **You may not use calculators**, books, notes, or any other material to help you. Please make sure your **phone is silenced and stowed** where you cannot see it.
- You may use any available space on the exam for scratch work. If you need more scratch paper, please ask one of the proctors.
- In each of the questions 2 through 4, you must **show your work** to receive full credit. Please write your solutions in the space below the questions. You must indicate if you go over the pages.

Question	Points	Score
1	9	
2	9	
3	11	
4	11	
Total	40	

1. Each of the following multiple choice questions has exactly one correct answer. Indicate your response by **marking the corresponding box** in each of the questions.

   You do **not** need to show any scratch work, and we will **not** grade your scratch work. No partial points will be given.

   (a) (1 pt) For $\mathcal{R} = [2,4] \times [-1,1]$, the integral $\iint_{\mathcal{R}} x \, \mathrm{d}A$ is equal to:

   A. $\int_2^4 \int_{-1}^1 x \, \mathrm{d}x \, \mathrm{d}y$

   B. $\int_{-1}^1 \int_2^4 y \, \mathrm{d}x \, \mathrm{d}y$

   C. $\int_2^4 \int_{-1}^1 x \, \mathrm{d}y \, \mathrm{d}x$

   D. $\int_{-1}^1 \int_2^4 x \, \mathrm{d}y \, \mathrm{d}x$

   Your response:

A	B	C	D

   (b) (1 pt) If $\mathcal{R} = [-1,1] \times [0,2]$, the integral $\iint_{\mathcal{R}} (1+x) \, \mathrm{d}A$ is equal to:

   A. 0

   B. 1

   C. 4

   D. 9

   Your response:

A	B	C	D

   (c) (1 pt) If $\mathcal{D}$ is the domain bounded by the curves $y = x^2$ and $y = x + 2$, then $\mathcal{D}$ has the description:

   A. $-1 \leq x \leq 2, \quad x+2 \leq y \leq x^2$

   B. $1 \leq y \leq 4, \quad y-2 \leq x \leq \sqrt{y}$

   C. $0 \leq y \leq 4, \quad y-2 \leq x \leq \sqrt{y}$

   D. $-1 \leq x \leq 2, \quad x^2 \leq y \leq x+2$

   Your response:

A	B	C	D

(d) (1 pt) If $\mathcal{B} = [0,1] \times [0,1] \times [0,1]$, the integral $\iiint_{\mathcal{B}} x \sin(\pi z)\, \mathrm{d}V$ is equal to:

A. $\frac{1}{\pi}$

B. $\frac{2}{\pi}$

C. $\frac{1}{2\pi}$

D. 0

Your response:

A	B	C	D

(e) (1 pt) The integral $\iiint_{\mathcal{W}} 3z^2\, \mathrm{d}V$ over the domain $\mathcal{W}$ defined by $x^2 + y^2 \leq 2$ and $-1 \leq z \leq 1$ is equal to:

A. 2π

B. 4π

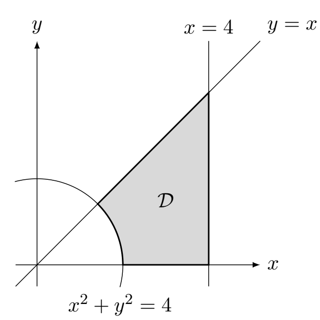
C. 6π

D. 12π

Your response:

A	B	C	D

(f) (1 pt) If $\mathcal{D}$ is the shaded region as in the figure to the right, then after changing to polar coordinates, the integral $\iint_{\mathcal{D}} x\, \mathrm{d}A$ becomes:



A. $\int_0^{\pi/4} \int_2^{4\sqrt{2}} r \cos\theta\, \mathrm{d}r\, \mathrm{d}\theta$

B. $\int_0^{\pi/4} \int_2^{4\sec\theta} r \cos\theta\, \mathrm{d}r\, \mathrm{d}\theta$

C. $\int_0^{\pi/4} \int_2^{4\sqrt{2}} r^2 \cos\theta\, \mathrm{d}r\, \mathrm{d}\theta$

D. $\int_0^{\pi/4} \int_2^{4\sec\theta} r^2 \cos\theta\, \mathrm{d}r\, \mathrm{d}\theta$

Your response:

A	B	C	D

(g) (1 pt) The image of the line $v = 2$ under the map $(x, y) = G(u, v) = (2u - v, 2u + 3v)$ is equal to

A. $y = 2$
B. $y = x + 8$
C. $y = 2x + 6$
D. $y = 8 - x$

Your response:

A	B	C	D

(h) (1 pt) The Jacobian of $G(u, v) = (uv^2, u^2/v)$ at $(u, v) = (2, 1)$ is equal to

A. -20
B. -12
C. 4
D. 16

Your response:

A	B	C	D

(i) (1 pt) If $\mathcal{D}$ is the image of $\mathcal{R} = [0, 1] \times [1, 2]$ under the map $G(u, v) = (uv^2, u^2/v)$, then the integral $\iint_{\mathcal{D}} ye^{xy^2}\, dx\, dy$ is equal to

A. $-\iint_{\mathcal{R}} 5u^3v^2e^{u^5}\, du\, dv$
B. $-\iint_{\mathcal{R}} \frac{5u^4e^{u^5}}{v}\, du\, dv$
C. $\iint_{\mathcal{R}} 5u^3v^2e^{u^5}\, du\, dv$
D. $\iint_{\mathcal{R}} \frac{5u^4e^{u^5}}{v}\, du\, dv$

Your response:

A	B	C	D

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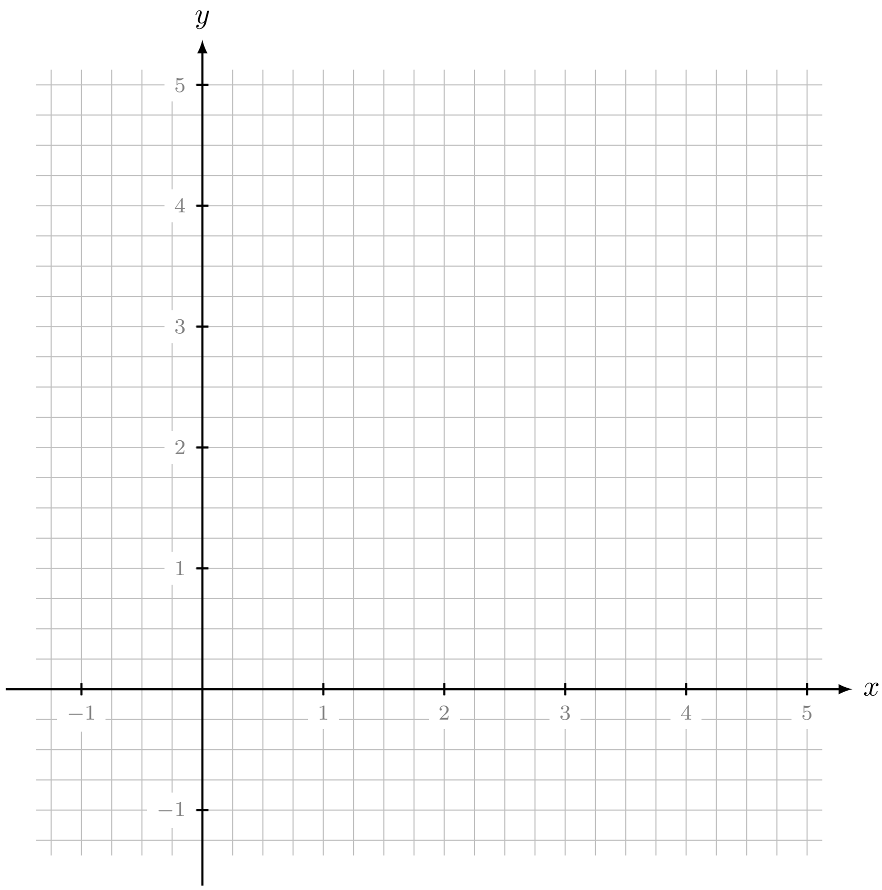
2. In this problem, we will compute the integral

$$I = \int_0^4 \int_{\sqrt{y}}^2 \sqrt{x^2 + 3y}\,\mathrm{d}x\,\mathrm{d}y$$

(a) (2 pts) Let $\mathcal{D}$ be the domain of integration of I. Note that $\mathcal{D}$ is a horizontally simple region described by $0 \le y \le 4, \ \sqrt{y} \le x \le 2$. Sketch the region $\mathcal{D}$ on the graph provided:



(b) (3 pts) Express $\mathcal{D}$ as a vertically simple region, i.e., in the form $a \le x \le b, \ g_1(x) \le y \le g_2(x)$.

(c) (4 pts) Evaluate I by changing the order of integration.

3. Consider the region $\mathcal{D}$ in the xy-plane described by the inequalities

$$x^2 + y^2 \leq 1 \qquad \text{and} \qquad y \geq 1 - \sqrt{3}\, x.$$

(a) (3 pts) Express $\mathcal{D}$ as a radially simple region, i.e., in the form $\theta_1 \leq \theta \leq \theta_2,\ r_1(\theta) \leq r \leq r_2(\theta)$ in polar coordinates.

(b) (4 pts) Integrate the function $f(x, y) = (x^2 + y^2)^{-3/2}$ over $\mathcal{D}$.

(c) (4 pts) Find the area of $\mathcal{D}$.

Hint: You might find the identity $\int \frac{1}{(\sqrt{3}\cos\theta+\sin\theta)^2}\,\mathrm{d}\theta = -\frac{\cos\theta}{\sqrt{3}\cos\theta+\sin\theta} + C$ useful.

4. Let $\mathcal{W}$ denote the region defined by $r^2 + (z-2)^2 \leq 4$ and $0 \leq \theta \leq 2\pi$ in cylindrical coordinates.

   (a) (1 pt) Express $\mathcal{W}$ in rectangular coordinates (x, y, z).

   (b) (3 pts) Express $\mathcal{W}$ in spherical coordinates (ρ, θ, ϕ).

   Hint: You may use the fact that spherical coordinates and cylindrical coordinates are related by $(r, \theta, z) = (\rho \sin \phi, \theta, \rho \cos \phi)$.

(c) (4 pts) The average distance from the origin of all points inside $\mathcal{W}$ is given by

$$\frac{1}{V}\iiint_{\mathcal{W}} \sqrt{x^2+y^2+z^2}\,\mathrm{d}V$$

where $V = \iiint_{\mathcal{W}} 1\,\mathrm{d}V$ is the volume of $\mathcal{W}$. Evaluate the above integral using spherical coordinates.

Hint: V can be easily computed by recognizing $\mathcal{W}$ as a well-known geometric object.

(d) (3 pts) Evaluate the integral

$$\iiint_{\mathcal{W}} \frac{1}{\sqrt{x^2+y^2}}\,\mathrm{d}V.$$

Hint: You might find the identity $\int_0^{\pi/2} \cos^2\theta\,\mathrm{d}\theta = \pi/4$ useful.

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