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- Some exercises are taken from R.V. Hogg, E. Tanis, D. Zimmerman, *Probability and Statistical Inference*, 10th Ed., Pearson.

3.4 Additional Models

1. Let X be a random variable of the mixed type having the CDF

$$F(x) = \begin{cases} 0, & x < -1, \\ \dfrac{x+2}{3}, & -1 \le x < 1, \\ 1, & 1 \le x. \end{cases}$$

Determine the indicated probabilities.

(a) $P(X < 0)$

(b) $P(X < -1)$

(c) $P(X \le -1)$

(d) $P(-1 \le X < \frac{1}{2})$

(e) $P(-1 < X < 1)$

2. Let X has the CDF of the form

$$F(x) = \begin{cases} 0, & x < 0, \\ 1 - \left(\frac{2}{5}\right) e^{-2x}, & 0 \le x. \end{cases}$$

Do the following:

(a) Compute $E(X)$.

(b) Compute $\mathrm{Var}(X)$.

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3. The weekly gravel demand X (in tons) follows the exponential distribution with mean $\theta = 8$. However, the owner of the gravel pit can produce at most only six tons of gravel per week. Compute the expected value of the tons sold per week by the owner.

4.1 Bivariate Distributiosn of the Discrete Type

1. For each of the following functions, determine the constant c so that $p(x, y)$ satisfies the conditions of being a joint PMF for two discrete random variables:

(a) $p(x,y) = c(2x + y), \qquad x = 1, 2,\ y = 1, 2, 3.$

(b) $p(x,y) = c, \qquad x$ and y are positive integers satisfying $x + y \leq 6$.

(c) $p(x,y) = c\left(\frac{1}{3}\right)^x\left(\frac{1}{5}\right)^y, \qquad x$ and y are positive integers.

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2. Let

$$p(x,y) = \frac{xy^2}{30}$$

be the joint PMF of discrete random variables X and Y with the support

$$S = \{(x,y) : x = 1,2,3,\ y = 1,2\}.$$

Do the following:

(a) Find $p_X(x)$, the marginal PMF of X.

(b) Find $p_Y(y)$, the marginal PMF of Y.

(c) Are X and Y independent or dependent? Why or why not?

(d) Find $P(X < Y)$.

(e) Find $P(X + Y = 3)$.

(f) Compute $E(X - 2Y)$

3. Let

$$p(x,y) = \frac{xy^2}{30}$$

be the joint PMF of discrete random variables X and Y with the support

$$S = \{(1,2),(1,3),(2,1),(3,1),(3,2)\}.$$

Do the following:

(a) Find $p_X(x)$, the marginal PMF of X.

(b) Find $p_Y(y)$, the marginal PMF of Y.

(c) Are X and Y independent or dependent? Why or why not?

(d) Find $P(Y = 2X)$.

(e) Find $P(X \leq 3 - Y)$.

(f) Compute $E(XY)$.

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4. A particle starts at $(0, 0)$ and moves in one-unit independent steps with equal probabilities of $1/4$ in each of the four directions: north, south, east, and west. Let S equal the east-west position and T the north-south position after three steps.

(a) Define the joint pmf of S and T. On a two-dimensional graph, give the probabilities of the joint PMF and the marginal PMFs (similar to Figure 4.1-1).

(b) Let $X = S + 3$ and $Y = T + 3$. Give the marginal PMFs of X and Y.

5. Let X and Y be independent random variables taking only integer values. Let

$$Z = X + Y,$$

which also takes only integer values. Its PMF can be computed by the **convolution formula**: for any integer z,

$$\begin{aligned} p_Z(z) = P(Z = z) &= P(X + Y = z) \\ &= \sum_{x=-\infty}^{\infty} P(X = x,\ X + Y = z) \qquad (1) \\ &= \sum_{x=-\infty}^{\infty} P(X = x,\ Y = z - x) \qquad (2) \\ &= \sum_{x=-\infty}^{\infty} P(X = x)P(Y = z - x) \qquad (3) \\ &= \sum_{x=-\infty}^{\infty} p_X(x)p_Y(z - x). \end{aligned}$$

Do the following:

(a) Explain why the steps (1), (2), and (3) hold.

(b) Let X and Y be independent and have geometric distribution with parameter p. By computing the PMF, show that $X + Y$ has a negative binomial distribution with parameter $r = 2$ and p.

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4.2 The Correlation Coefficient

1. Let the random variables X and Y have the joint PMF of the form

$$p(x,y) = \frac{x+y}{21}, \qquad x = 1,2,\ y = 1,2,3.$$

They satisfy

$$\mu_X = \frac{11}{7}, \qquad \sigma_X^2 = \frac{12}{7^2},$$
$$\mu_Y = \frac{46}{21}, \qquad \sigma_Y^2 = \frac{278}{21^2}.$$

Find the covariance $\text{Cov}(X,Y)$ and the correlation coefficient ρ. Are X and Y independent or dependent?

2. Roll a fair six-sided die three times. Let

$$X = [\text{\# of times you roll a 1, 2, or 3}],$$
$$Y = [\text{\# of times you roll a 5}].$$

Do the following:

(a) Find $E(X)$ and $\text{Var}(X)$.
Hint: What is the distribution of X?

(b) Find $E(Y)$ and $\text{Var}(Y)$.
Hint: What is the distribution of Y?

(c) Find $\text{Cov}(X,Y)$.

(d) Find ρ. Does the fact that $\rho < 0$ makes sense to you intuitively?

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3. The joint PMF of X and Y is

$$p(x, y) = \frac{1}{6}, \qquad x \text{ and } y \text{ are non-negative integers satisfying } x + y \leq 6.$$

Do the following:

(a) Sketch the support of X and Y.

(b) Record the marginal PMFs of X and Y in the "margins".

(c) Compute ρ.

(d) Find the least squares regression line and draw it on your figure.

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4. Let the joint PMF of X and Y be

$$p(x,y) = \frac{1}{4}, \qquad (x,y) \in S = \{(-2,0),(0,1),(0,-1),(2,0)\}.$$

(a) Are X and Y independent?

(b) Calculate $\mathrm{Cov}(X,Y)$ and ρ.

5. Let X and Y be independent random variables of the discrete type. Simplify

$$\mathrm{Cov}(X+2Y, 3X-Y).$$

6. If X and Y are independent random variables of the discrete type, then we learned that

$$E[u(X)v(Y)] = E[u(X)]E[v(Y)]$$

holds as long as all of the expectations exist. Applying this to the choice $u(x) = v(x) = e^{tx}$, we get

$$M_{X+Y}(t) = E[e^{t(X+Y)}] = E[e^{tX}]E[e^{tY}] = M_X(t)M_Y(t).$$

Using this observation, do the following:

(a) Let X and Y be independent. Suppose that X is $b(m,p)$ and Y is $b(n,p)$. By examining the MGF, show that $X+Y$ is $b(m+n,p)$.

(b) Let X and Y be independent and have Poisson distributions with rates λ_1 and λ_2, respectively. By examining the MGF, show that $X+Y$ has a Poisson distribution with rate $\lambda_1 + \lambda_2$.