Mathe 170E Lecture 1 Winter 2020	Homework 5	Due February 7, in class

Name:	**Student ID:**

1. You just rented a large house and the realtor gave you 5 keys, one for each of the 5 doors of the house. Unfortunately, all keys look identical. so to open the front door, you try them at random.

(a) Find the PMF of the number of trials you will need to open the door, under the following alternative assumptions: (1) after an unsuccessful trial. you mark the corresponding key. so that you never try it again. and (2) at each trial you are equally likely to choose any key.

(b) Repeat part (a) for the case where the realtor gave you an extra duplicate key for each of the 5 doors.

2. An excellent free-throw shooter attempts several free throws until she misses.

(a) If $p = 0.9$ is her probability of making a free throw, what is the probability of having the first miss on the 13th attempt or later?

(b) If she continues shooting until she misses three, what is the probability that the third miss occurs on the 30th attempt?

3. Suppose an airport metal detector catches a person with metal 99% of the time. That is, it misses detecting a person with metal 1% of the time. Assume independence of people carrying metal. What is the probability that the first metal-carrying person missed (not detected) is among the first 50 metal-carrying persons scanned?

4. Flip a fair coin repeatedly. What is the probability that the number of coin flips until the second head occurs is at most 10?

Hint: This is equal to the probability that at least two heads occur in the first 10 flips.

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5. Two old gamblers alternate at rolling a fair 6-sided die until the first 6 comes up. Compute the probability of the first player winning.

6. One of four different prizes was randomly put into each box of a cereal. If a family decided to buy this cereal until it obtained at least one of each of the four different prizes, what is the expected number of boxes of cereal that must be purchased?

7. Let X have a Poisson distribution with a mean of 4. Find

(a) $P(2 \leq X \leq 5)$.

(b) $P(X \geq 3)$.

(c) $P(X \leq 3)$.

8. Customers arrive at a travel agency at a mean rate of 11 per hour. Assuming that the number of arrivals per hour has a Poisson distribution, give the probability that more than 10 customers arrive in a given hour.

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9. Flaws in a certain type of drapery material appear on the average of one in 150 square feet. If we assume a Poisson distribution, find the probability of at most one flaw appearing in 225 square feet.

10. With probability 0.001, a prize of $499 is won in the Michigan Daily Lottery when a $1 straight bet is placed. Let Y equal the number of $499 prizes won by a gambler after placing n straight bets. Note that Y has $b(n, 0.001)$ distribution. After placing $n = 2000$ $1 bets, the gambler is behind or even if $\{Y \leq 4\}$. Use the Poisson distribution to approximate $P(Y \leq 4)$ when $n = 2000$.

11. A certain type of aluminum screen that is 2 feet wide has, on the average, one flaw in a 100-foot roll. You purchase 6 of 50-foot rolls. Find the probability that none of them has flaws.

12. You go to a party with 500 guests. Using the Poisson approximation, find the probability that exactly one other guest has the same birthday as you. (For simplicity. exclude birthdays on February 29.)

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13. An airline always overbooks if possible. A particular plane has 95 seats on a flight in which a ticket sells for $300. The airline sells 100 such tickets for this flight.

(a) If the probability of an individual not showing up is 0.05, assuming independence, what is the probability that the airline can accommodate all the passengers ho do show up?

(b) If the airline must return the $300 price plus a penalty of $400 to each passenger that cannot get on the flight, what is the expected payout (penalty plus ticket refund) that the airline will pay?

References

The following problems are taken from R.V. Hogg, E. Tanis, D. Zimmerman, *Probability and Statistical Inference*, 10th Ed., Pearson.

- #2: Exercise 2.6-1
- #3: Exercise 2.6-4
- #6: Exercise 2.6-9
- #7: Exercise 2.7-1
- #8: Exercise 2.7-3
- #9: Exercise 2.7-5
- #10: Exercise 2.7-7
- #13: Exercise 2.7-11

The following problem is taken from D.P. Bertsekas, J.N. Tsitsiklis, *Introduction to Probability*, 2nd Ed., Athena Scientific.

- #1: Problem 2.7