Homework 2

- Homework 2 is due January 17 in class.
- Exercises are taken from R.V. Hogg, E. Tanis, D. Zimmerman, *Probability and Statistical Inference*, 10th Ed., Pearson.
- The starred problems will not be graded.

1.3 Conditional Probability

Exercises

2. The following table classifies 1456 people by their gender and by whether or not they favor a gun law.

	Male (S_1)	Female (S_2)	Totals
Favor (A_1)	392	649	1041
Oppose (A_2)	241	174	415
Totals	633	823	1456

Compute the following probabilities if one of these 1456 persons is selected randomly: **(a)** $P(A_1)$, **(b)** $P(A_1 \mid S_1)$, **(c)** $P(A_1 \mid S_2)$. **(d)** Interpret your answers to parts (b) and (c).

4. Two cards are drawn successively and without replacement from an ordinary deck of playing cards. Compute the probability of drawing

(a) Two hearts.

(b) A heart on the first draw and a club on the second draw.

(c) A heart on the first draw and an ace on the second draw.

Hint: In part (c), note that a heart can be drawn by getting the ace of hearts or one of the other 12 hearts.

5. Suppose that the alleles for eye color for a cer tain male fruit fly are (R, W) and the alleles for eye color for the mating female fruit fly are (R, W), where R and W represent red and white, respectively. Their offspring receive one allele for eye color from each parent.

(a) Define the sample space of the alleles for eye color for the offspring.

(b) Assume that each of the four possible outcomes has equal probability. If an offspring ends up with either two white alleles or one red and one white allele for eye color, its eyes will look white. Given that an offspring's eyes look white, what is the conditional probability that it has two white alleles for eye color?

8. An urn contains 17 balls marked LOSE and 3 balls marked WIN. You and an opponent take turns selecting a single ball at random from the urn without replacement. The person who selects the third WIN ball wins the game. It does not matter who selected the first two WIN balls.

(a) If you draw first, find the probability that you win the game on your second draw.

(b) If you draw first, find the probability that your opponent wins the game on his second draw.

(c) If you draw first, what is the probability that you win? *Hint:* You could win on your second, third, fourth, ..., or tenth draw, but not on your first.

(d) Would you prefer to draw first or second? Why?

10. A single card is drawn at random from each of six well-shuffled decks of playing cards. Let A be the event that all six cards drawn are different.

(a) Find $P(A)$.

(b) Find the probability that at least two of the drawn cards match.

16. Bowl A contains three red and two white chips, and bowl B contains four red and three white chips. A chip is drawn at random from bowl A and transferred to bowl B. Compute the probability of then drawing a red chip from bowl B.

1.4 Independent Events

Exercises not in the Textbook

N1*. Let A and B be events with $P(A) = 0$. Show that A and B are independent. *Hint:* Show that both $P(A \cap B)$ and $P(A)P(B)$ are zero.

This exercise tells that, to show whether A and B are independent, it suffices to check whether $P(A) = 0$ or $P(B \mid A) = P(B)$ holds.

N2*. Let A, B, and C be events. Show that the followings are equivalent:

(1) A, B, and C are mutually independent.

(2) A', B, and C are mutually independent.

(3) A', B', and C are mutually independent.

(4) A', B', and C' are mutually independent.

Hint: For the equivalence $(1) \Leftrightarrow (2)$, mimic the proof of Theorem 1.4-1. To show $(2) \Leftrightarrow (3)$, apply the equivalence $(1) \Leftrightarrow (2)$ to the events A', B, and C in place of A, B, and C. Likewise for $(3) \Leftrightarrow (4)$.

Exercises

1. Let A and B be independent events with $P(A) = 0.7$ and $P(B) = 0.2$. Compute **(a)** $P(A \cap B)$, **(b)** $P(A \cup B)$, and **(c)** $P(A' \cup B')$.

5. If $P(A) = 0.8$, $P(B) = 0.5$, and $P(A \cup B) = 0.9$, are A and B independent events? Why or why not?

6*. Show that if A, B, and C are mutually independent, then the following pairs of events are independent: A and $(B \cap C)$, A and $(B \cup C)$, A' and $B \setminus C$.

7. Each of three football players will attempt to kick a field goal from the 25-yard line. Let A_i denote the event that the field goal is made by player i, $i = 1, 2, 3$. Assume that A_1, A_2, A_3 are mutually independent and that $P(A_1) = 0.5$, $P(A_2) = 0.7$, $P(A_3) = 0.6$.

(a) Compute the probability that exactly one player is successful. *Hint:* This event is the union of the following mutually exclusive events:

$$A_1 \cap A_2' \cap A_3', \qquad A_1' \cap A_2 \cap A_3', \qquad \textit{and} \qquad A_1' \cap A_2' \cap A_3.$$

Now invoke Exercise N2* (see above) to compute the probabilities of these events.

(b) Compute the probability that exactly two players make a field goal (i.e., one misses).

10. Suppose that A, B, and C are mutually independent events and that $P(A) = 0.5$, $P(B) = 0.8$, and $P(C) = 0.9$. Find the probabilities of the following events:

(a) All three events occur.

(b) Exactly two of the three events occur.

(c) None of the events occurs.

13. An urn contains two red balls and four white balls. Sample successively five times at random and with replacement, so that the trials are independent.

(a) Compute the probability that no two balls drawn consecutively in the sequence of five balls have the same color.

(b) How would your answer to part (a) change if the sampling is without replacement?

19. Extend Example 1.4-6 to an n-sided die. That is, suppose that a fair n-sided die is rolled n independent times. A match occurs if side i is observed on the ith trial, $i = 1, 2, \ldots, n$.

(a) Show that the probability of at least one match is

$$1 - \left(\frac{n-1}{n}\right)^n = 1 - \left(1 - \frac{1}{n}\right)^n.$$

(b) Find the limit of this probability as n increases without bound.

1.5 Bayes' Theorem

Exercises

1. Bowl B_1 contains two white chips, bowl B_2 contains two red chips, bowl B_3 contains two white and two red chips, and bowl B_4 contains three white chips and one red chip. The probabilities of selecting bowl B_1, B_2, B_3, or B_4 are $1/2$, $1/4$, $1/8$, and $1/8$, respectively. A bowl is selected using these probabilities and a chip is then drawn at random. Find

(a) $P(W)$, the probability of drawing a white chip.

(b) $P(B_1 \mid W)$, the conditional probability that bowl B_1 had been selected, given that a white chip was drawn.

4. Assume that an insurance company knows the following probabilities relating to automobile accidents (where the second column refers to the probability that the policyholder has at least one accident during the annual policy period):

Age of Driver	Probability of Accident	Fraction of Company's Insured Drivers
16–25	0.05	0.10
26–50	0.02	0.55
51–65	0.03	0.20
66–90	0.04	0.15

A randomly selected driver from the company's insured drivers has an accident. What is the conditional probability that the driver is in the 16–25 age group?

10. Suppose we want to investigate the percentage of abused children in a certain population. To do this, doctors examine some of these children taken at random from that population. However, doctors are not perfect: They sometimes classify an abused child (A^+) as one not abused (D^-) or they classify a nonabused child (A^-) as one that is abused (D^+). Suppose these error rates are $P(D^- \mid A^+) = 0.08$ and $P(D^+ \mid A^-) = 0.05$, respectively; thus, $P(D^+ \mid A^+) = 0.92$ and $P(D^- \mid A^-) = 0.95$ are the probabilities of the correct decisions. Let us pretend that only 2% of all children are abused; that is, $P(A^+) = 0.02$ and $P(A^-) = 0.98$.

(a) Select a child at random. What is the probability that the doctor classifies this child as abused? That is, compute

$$P(D^+) = P(A^+)P(D^+ \mid A^+) + P(A^-)P(D^+ \mid A^-).$$

(b) Compute $P(A^- \mid D^+)$ and $P(A^+ \mid D^+)$.

(c) Compute $P(A^- \mid D^-)$ and $P(A^+ \mid D^-)$.

(d) Are the probabilities in (b) and (c) alarming? This happens because the error rates of 0.08 and 0.05 are high relative to the fraction 0.02 of abused children in the population.

11. At the beginning of a certain study of a group of persons, 15% were classified as heavy smokers, 30% as light smokers, and 55% as nonsmokers. In the five-year study, it was determined that the death rates of the heavy and light smokers were five and three times that of the nonsmokers, respectively. A randomly selected participant died over the five-year period; calculate the probability that the participant was a nonsmoker.