Homework 1

- Homework 1 is due January 10 in class.
- Exercises are taken from R.V. Hogg, E. Tanis, D. Zimmerman, *Probability and Statistical Inference*, 10th Ed., Pearson.

1.1 Properties of Probability

Exercises

3. Draw one card at random from a standard deck of cards. The sample space S is the collection of the 52 cards. Assume that the probability set function assigns $1/52$ to each of the 52 outcomes. Let

$$\begin{aligned} A &= \{x : x \text{ is a jack, queen, or king}\}, \\ B &= \{x : x \text{ is a 9, 10, or jack and } x \text{ is red}\}, \\ C &= \{x : x \text{ is a club}\}, \\ D &= \{x : x \text{ is a diamond, a heart, or a spade}\}. \end{aligned}$$

Find

(a) $P(A)$,
(b) $P(A \cap B)$,
(c) $P(A \cup B)$,
(d) $P(C \cup D)$,
(e) $P(C \cap D)$.

5. Consider the trial on which a 3 is first observed in successive rolls of a six-sided die. Let A be the event that 3 is observed on the first trial. Let B be the event that at least two trials are required to observe a 3. Assuming that each side has probability $1/6$, find

(a) $P(A)$,
(b) $P(B)$,
(c) $P(A \cup B)$.

10. Prove Theorem 1.1-6.

1.2 Methods of Enumeration

Exercises

3. How many different license plates are possible if a state uses

(a) Two letters followed by a four-digit integer (leading zeros are permissible and the letters and digits can be repeated)?
(b) Three letters followed by a three-digit integer? (In practice, it is possible that certain "spellings" are ruled out.)

7. In a state lottery, four digits are drawn at random one at a time with replacement from 0 to 9. Suppose that you win if any permutation of your selected integers is drawn. Give the probability of winning if you select

(a) $6, 7, 8, 9$.
(b) $6, 7, 8, 8$.
(c) $7, 7, 8, 8$.
(d) $7, 8, 8, 8$.

10. Pascal's triangle gives a method for calculating the binomial coefficients; it begins as follows:

$$\begin{array}{ccccccccccc} & & & & & 1 & & & & & \\ & & & & 1 & & 1 & & & & \\ & & & 1 & & 2 & & 1 & & & \\ & & 1 & & 3 & & 3 & & 1 & & \\ & 1 & & 4 & & 6 & & 4 & & 1 & \\ 1 & & 5 & & 10 & & 10 & & 5 & & 1 \\ & & \vdots & & \vdots & \vdots & \vdots & & \vdots & & \end{array}$$

The nth row of this triangle gives the coefficients for $(a+b)^{n-1}$. To find an entry in the table other than a 1 on the boundary, add the two nearest numbers in the row directly above. The equation

$$\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1},$$

called **Pascal's equation**, explains why Pascal's triangle works. Prove that this equation is correct.

17. A poker hand is defined as drawing five cards at random without replacement from a deck of 52 playing cards. Find the probability of each of the following poker hands:

(a) Four of a kind (four cards of equal face value and one card of a different value).
(b) Full house (one pair and one triple of cards with equal face value).
(c) Three of a kind (three equal face values plus two cards of different values).
(d) Two pairs (two pairs of equal face value plus one card of a different value).
(e) One pair (one pair of equal face value plus three cards of different values).