PRACTICE FINAL EXAM

June 13, 2002

Instructions.

Please show your work. You will receive little or no credit for an answer not accompanied by appropriate explanations, even if the answer is correct. If you have a question about a particular problem, please raise your hand and one of the proctors will come and talk to you.

Calculators or computers of any kind are not allowed. You are not allowed to consult any other materials of any kind, including books, notes and your neighbors. You will find a list of some useful formulas on page 2 of the exam.

At the end of the exam, please hand the exam paper to your TA. Please be prepared to show your university ID upon request.

Name:_____ Student ID:_____

Section:_____

#1	#2	#3	#4	#5	#6	#7
#8	#9	#10	#11	#12	#13	Total

$$\cos^2 t + \sin^2 t = 1 \qquad \sin 2t = 2 \sin t \cos t$$

$$\sin^2 \frac{t}{2} = \frac{1 - \cos t}{2} \qquad \cos^2 \frac{t}{2} = \frac{1 + \cos t}{2}$$

$$\cos 2t = \cos^2 t - \sin^2 t \qquad \frac{\partial f}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x}$$

$$L = \int_a^b \| \overrightarrow{r}' \| dt \qquad \frac{d}{dt} \langle f, g, h \rangle = \langle f', g', h' \rangle$$

$$A = \int_a^b \frac{1}{2} [f(\theta)]^2 d\theta \qquad \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\overrightarrow{v} \cdot \overrightarrow{w} = \| \overrightarrow{v} \| \| \overrightarrow{w} \| \cos \theta \qquad \| \overrightarrow{v} \times \overrightarrow{w} \| = \| \overrightarrow{v} \| \| \overrightarrow{w} \| |\sin \theta|$$

$$\overrightarrow{T}(t) = \frac{\overrightarrow{r}'(t)}{\| \overrightarrow{r}'(t) \|} \qquad \kappa = \frac{\| \overrightarrow{T}'(t) \|}{\| \overrightarrow{r}'(t) \|}$$

Problem 1. Find an equation of the plane through the points $(1, 1, 1)$, $(0, 1, 1)$ and $(-1, -1, -1)$.

Problem 2. (a) Prove that the vectors $\langle 1, 4, 7\rangle$, $\langle 2, 5, 8\rangle$ and $\langle 3, 6, 9\rangle$ are parallel to the same plane. (b) Find an equation of such a plane.

Problem 3. Let Cbe the parametric curve $x = \sin t$, $y = 3\cos t$. Sketch this curve. Find all points on the curve at which the tangent line to the curve is parallel to the line $y = x$.

Problem 4. Find an equation in polar coordinates that describes the line $y = 3x + 1$.

Problem 5. Let $f(x,y) = \frac{x}{y}$. (a) Determine the domain and range of f. (b) Sketch enough level lines of f to give an idea of how the level lines look like. (c) Does the limit $\lim_{(x,y)\to(0,0)} f(x,y)$ exist?

Problem 6. Evaluate the following limit, or show that the limit does not exist

$$\lim_{(x,y)\to(0,0)} \frac{\sin(x+y)}{x+y}.$$

Solution.

Problem 7. Let $f(x, y) = e^{-(x^2+y^2)}$. Compute the following partial derivatives of f:

$$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial y \partial x}.$$

Problem 8. Find the maximum value of xyz if $x+y+z = 1$ and $x, y, z \geq 0$.

Problem 9. Consider the polar curve

$$r = 2 - \sin 3\theta.$$

(a) Sketch the curve.
(b) Find the area between the x-axis and the part of the curve which lies below the x axis.

Problem 10. Find all points on the surface $x^2 + 3y + z^2 = 1$ at which the tangent plane is parallel to the xz-plane.

Problem 11. Let $f(x, y) = \sin xy$. Find the minimum and maximum values of f in the region $x^2 + y^2 \leq 1$.

Problem 12. Find all critical points of

$$f(x, y) = x^2 + y^2 + y^3 + 3xy^2$$

and classify them as local minima, local maxima, or saddle points.