

# Math 31B Quiz 7A.

December 8, 2005

Please enter your last name, first name and student ID on the test card provided to you. Please make sure that you start at the leftmost bubble for each field that you fill in.

You are taking quiz 7A. Please fill in your answers in **bubbles 1-4 on the test card**. Putting any marks in any other bubble will result in an automatic zero for the entire quiz. Please call for help if you have any questions!

1. Which of the following is the Taylor series for  $e^{x^3}$ ?

- (a)  $1 + x^3/3! + x^6/6! + \dots$  (b)  $1 + x^3 + x^3/2 + \dots$  (c)  $1 + x + x^2/2! + \dots$  (d) No Taylor series exists (e) None of the above

2. Let  $f(x) = \frac{e^{x^2} - 1}{x^n}$ ,  $n \geq 0$ . For which  $n$  does the limit  $\lim_{x \rightarrow 0} f(x)$  exist? Hint: expand the numerator and denominator as Taylor series and consider the leading term.

- (a) any  $n$  (b)  $n \geq 2$  (c)  $n \leq 2$  (d)  $n = 1$  only (e) None of the above

3. Let  $f(x) = \sum a_n x^n$  be a power series. Assume that  $f$  converges for all  $x$  and that  $f$  is odd (i.e.,  $f(x) = -f(-x)$ ). What can be said about the coefficients  $a_n$ ?

- (a)  $a_0 = 0$  (b)  $a_n = 0$  for  $n$  even (c)  $a_4 = 0$  (d) all of the above (e) None of the above

4. Let  $f(x) = \frac{1}{1 - \sin x}$ . What is the value of  $f''(0)$ ? Hint: find the Taylor series for  $f(x)$  first.

- (a) 1 (b) 2 (c) -1 (d) -2 (e) None of the above

$$\begin{aligned} \frac{1}{1 - \sin x} &= 1 + \sin x + \sin^2 x + \dots \\ &= 1 + \left(x - \frac{x^3}{3!} + \text{higher}\right) + \left(x - \frac{x^3}{3!} + \text{higher}\right)^2 + \dots \\ &= 1 + x - \frac{x^3}{3!} + \text{higher} + x^2 + \text{higher} \\ &= 1 + x + x^2 + \text{higher}. \quad \text{So } f'' = 2 + \text{higher} \\ &\quad f''(0) = 2. \end{aligned}$$