Math 31B Quiz 6B.

December 1, 2005

1. Determine if the series $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^3+1}$ is:

   (a) absolutely convergent $\boxed{\text{(b)}}$ conditionally convergent (c) divergent
   Solution — do the limit comparison to $1/n$ to see that the series of asbolute values is divergent, so the series is not absolutely convergent. Now use the alternating series test to conclude that the series is convergent, hence conditionally convergent.

2. Determine the radius of convergence of the following power series: $\sum_{n=0}^{\infty} n^2 x^n$.

   (a) 2 $\boxed{\text{(b)}}$ 1 (e) None of the above
   (c) 0 (d) ∞
   Solution — use the ratio test to conclude that the radius is 1.

3. Assume that $f(x)$ is represented by a power series, $f(x) = a_0 + a_1 x + a_2 x^2 + \cdots$. Assume also that $f(0) = 0$ and $f''(0) = 4$. What can be said about the power series?

   (a) $a_0 = 1$ and $a_2 = 4$ (b) $a_0 = 0$ and $a_1 = 1$ (e) None of the above
   (c) cannot be determined $\boxed{\text{(d)}}$ $a_0 = 0$ and $a_2 = 2$
   Solution — note that $f(0) = a_0$ and $f''(0) = 2a_2$. Hence $a_0 = 0$ and $2a_2 = 4$.

4. Assume that $\sum_{n=1}^{\infty} a_n x^n$ is a power series, which converges at $x = 2$. Which of the following must happen?

   (a) The series converges at $x = -2$ (b) The series converges at $x = 2.5$ (e) None of the above
   $\boxed{\text{(c)}}$ The series converges at $x = -1$. (d) All of the above
   Solution — if R is the radius of convergence of the series, then the series is divergent outside of $[-R, R]$. Since it is convergent at 2, it should be that 2 is not outside $[-R, R]$, hence $R \geq 2$. So the series must converge at least on $(-2, 2)$, but it may diverge at -2. Thus (c) is the only thing that must happen.