Math 31B Quiz 5A.

November 10, 2005

Please enter your last name, first name and student ID on the test card provided to you. Please make sure that you start at the leftmost bubble for each field that you fill in.

You are taking quiz 5A. Please fill in your answers in **bubbles 1–4 on the test card.** Putting any marks in any other bubble will result in an automatic zero for the entire quiz. Please call for help if you have any questions!

1. The arc of the parabola $y = \sqrt{x}$ from $(1,1)$ to $(4,2)$ is rotated about the x-axis. What is the area of the resulting surface? $A = 2\pi\int_{x=1}^{x=4} y\,ds = 2\pi\int_1^4 \sqrt{x}\sqrt{1+\frac{1}{4x}} = \pi\int_1^4 \sqrt{4x+1}\,dx$

   (a) $\frac{\pi}{6}(17\sqrt{17}-5\sqrt{5})$ (b) $\frac{\pi}{6}(\sqrt{17}-\sqrt{15})$ (e) None of the above
   (c) $\frac{\pi}{6}(\sqrt{3}+\sqrt{17})$ (d) $\frac{\pi}{6}(\sqrt{6}-\sqrt{2})$

   $= \frac{1}{4}\pi\frac{2}{3}(4x+1)^{3/2}\Big|_1^4 = \frac{\pi}{6}(17\sqrt{17}-5\sqrt{5})$.

2. Let $f(x) = 1$. Use the trapezoidal rule with $n = 1,234,567$ to compute an approximation to the integral $\int_0^1 f(x)dx$. [Hint: think!]

   (a) 1 (b) 0 (e) None of the above
   (c) 2 (d) 1.2345678910

   Approx. is exact in this case, so $T_n = \int_0^1 f(x)dx = 1$.

3. Let $a_n = \ln(n+1) - \ln(n)$. What is the limit of the sequence $\{a_n\}$?

   (a)1 (b) 0 (e) None of the above
   (c) doesn't exist (d) -1

   $\ln(n+1) - \ln(n) = \ln\left(\frac{n+1}{n}\right)$. Since $\frac{n+1}{n} \to 1$ as $n\to\infty$, $\ln(n+1) - \ln n \to \ln 1 = 0$.

4. Let $a_n = n\sin\frac{1}{n}$. What is the limit of the sequence $\{a_n\}$?

   (a) 0 (b) doesn't exist (e) None of the above
   (c) 1 (d) 2

Let $f(x) = x\sin\frac{1}{x}$. By l'Hospital's rule,

$$\lim_{x\to\infty} x\sin\frac{1}{x} = \lim_{x\to\infty}\frac{\sin\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty}\frac{-\frac{1}{x^2}\cos\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to\infty}\cos\frac{1}{x} = \cos 0 = 1.$$

[or: let $y = \frac{1}{x}$, so as $x\to\infty$, $y\to 0^+$; then

$$\lim_{x\to\infty} x\sin\frac{1}{x} = \lim_{y\to 0}\frac{\sin y}{y} = 1].$$