

# Math 31B Quiz 3A.

October 20, 2005

Please enter your last name, first name and student ID on the test card provided to you. Please make sure that you start at the leftmost bubble for each field that you fill in.

You are taking quiz 3A. Please fill in your answers in **bubbles 1-4 on the test card**. Putting any marks in any other bubble will result in an automatic zero for the entire quiz. Please call for help if you have questions!

$$u = \ln x \quad dv = dx; \quad du = 1/x \, dx, \quad v = x$$

1. Evaluate  $\int_1^e \ln x \, dx = x \ln x / e - \int_1^e 1/x \cdot x \, dx = e - (e-1) = 1.$

- (a) 1 (b) e (c) None of the above  
(d) -1 (e) 1/e

2. Let  $f(x) = \sin x$ . What is the average value of  $f$  over the interval  $0, \pi$ ?

(a) 1 (b) 0 (c) None of the above  
(d)  $2/\pi$  (e)  $\pi/2$   $\frac{1}{\pi-0} \int_0^\pi \sin x \, dx = \frac{1}{\pi} (-\cos x) \Big|_0^\pi$

3. Evaluate  $\int_0^{2\pi} \sin^4 x \, dx = \int_0^{2\pi} \left( \frac{1 - \cos 2x}{2} \right)^2 dx = \frac{1}{\pi} (1+1) = 2/\pi$   
(a)  $\pi/2$  (b)  $3\pi/4$  (c) None of the above  
(d)  $1 + \pi/2$  (e)  $1/2$   $= \frac{1}{4} \int_0^{2\pi} (1 - 2\cos 2x + \cos^2 2x) dx$

4. Evaluate  $\int_0^{2\pi} \sin x \sin 3x \, dx$

- (a)  $\pi/4$  (b)  $\pi/2$  (c) None of the above  
(d) 0 (e)  $\sqrt{2}$

$$\sin x \sin 3x =$$

$$= \frac{1}{2} (\cos(x+3x) - \cos(x-3x))$$

$$\int_0^{2\pi} \sin x \sin 3x \, dx$$

$$= -\frac{1}{2} \int_0^{2\pi} \cos(4x) - \cos(-2x) \, dx$$

$$= 0.$$

$$\begin{aligned} &= \frac{2\pi}{4} - \frac{1}{4} \int_0^{2\pi} 2\cos 2x \, dx + \int_0^{2\pi} \frac{\cos^2 2x}{4} \, dx \\ &= \frac{2\pi}{4} + \int_0^{2\pi} \frac{1 + \cos 4x}{8} \, dx \\ &= \frac{2\pi}{4} + \frac{2\pi}{8} + \int_0^{2\pi} \frac{\cos 4x}{8} \, dx \\ &= \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}. \end{aligned}$$