MATH 31B LECTURE 1 AND 3
2ND MIDTERM VERSION A

Key ??

Please note: Show your work. Correct answers not accompanied by sufficent explanations will receive little or no credit. Please call one of the proctors if you have any questions about a problem. No calculators, computers, PDAs, cell phones, or other devices will be permitted. *If you have a question about the grading or believe that a problem has been graded incorrectly, you must bring it to the attention of your professor within 2 weeks of the exam.*

#1	#2	#3	#4	#5	#6	Total

Section 1__ 3__ meets: Tuesday Thursday

TA name: ______________

SID:________________

Name: Liu, Chih-Chung

Problem 1. (Multiple choice, 10 pts) Evaluate the integral

$$\int_0^{e-1} \frac{2x^2+x+3}{(x^2+1)(x+1)}dx$$

Indicate your answer in the box below:

$\boxed{B}$ (a) $-\tan^{-1}(e-1)+2$; (b) $\tan^{-1}(e-1)+2$; (c) $\tan^{-1}(e-1)+4$ (d) $\tan^{-1}(e-1)-4$; (e) None of the above.

$$\frac{2x^2+x+3}{(x^2+1)(x+1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+1}$$

$$\Rightarrow 2x^2+x+3 = Ax^2+Ax+Bx+B+Cx^2+C$$
$$= (A+C)x^2+(A+B)x+B+C$$

$$\Rightarrow \begin{cases} A+C=2 \\ A+B=1 \\ B+C=3 \end{cases} \Rightarrow A=0,\ B=1,\ C=2$$

$$\& \quad \frac{2x^2+x+3}{(x^2+1)(x+1)} = \frac{1}{x^2+1} + \frac{2}{x+1}$$

$$\therefore \int_0^{e-1} \frac{2x^2+x+3}{(x^2+1)(x+1)}dx = \int_0^{e-1}\frac{1}{x^2+1}dx + 2\int_0^{e-1}\frac{1}{x+1}dx$$
$$= \tan^{-1}x\Big|_0^{e-1} + 2\ln(x+1)\Big|_0^{e-1}$$
$$= \tan^{-1}(e-1)+2$$

//

Problem 2. (Multiple choice, 10 pts) Find the length of the curve $y = \frac{2}{3}x^{3/2}$, $0 \le x \le 3$.

$\boxed{B}$ (a) ∞; (b) $\frac{14}{3}$; (c) $\frac{(3\sqrt{3}-1)}{2}$; (d) $\frac{2}{3}(3\sqrt{3}-1)$; (e) None of the above.

$$y' = \sqrt{x}$$

$$\int_0^3 \sqrt{1+(y')^2}\,dx$$

$$= \int_0^3 \sqrt{1+x}\,dx$$

$$= \frac{2}{3}\sqrt{(1+x)^3}\Big|_0^3$$

$$= \frac{2}{3}(8-1) = \frac{14}{3} \quad //$$

Problem 3. (Multiple choice, 10 pts) Let the sequence $\{a_n\}$ be defined by: $a_n = n^{1/n}$. Find the limit $\lim_{n\to\infty} a_n$, if the limit exists. Indicate your answer in the box below:

$\boxed{B}$ (a) 0; (b) 1; (c) π; (d) limit does not exist; (e) None of the above.

let $f(x) = x^{\frac{1}{x}}$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} x^{\frac{1}{x}}$$

let $y = x^{\frac{1}{x}} \Rightarrow \ln y = \frac{1}{x} \ln x$

$$\lim_{x\to\infty} (\ln y) = \lim_{x\to\infty} \frac{\ln x}{x}$$

$$\overset{\text{L'Hospital}}{=} \lim_{x\to\infty} \frac{1}{x} = 0$$

$\therefore$

$$\lim_{x\to\infty} y = e^0 = 1$$

$$\Rightarrow a_n \to 1$$

Problem 4. (25 pts) Consider the series $\sum_{n=2}^{\infty} \frac{2}{3^n}$.
(a) Determine if the series is convergent;
(b) If the series is convergent, find its limit.

$$\sum_{n=2}^{\infty} \frac{2}{3^n} = 2 \sum_{n=2}^{\infty} \frac{1}{3^n}$$

Geometric series with $r = \frac{1}{3}$, $a_1 = \cancel{\frac{1}{3}}\ \frac{1}{9}$

$$= 2 \frac{\frac{1}{9}}{1 - \frac{1}{3}}$$

$$= 2 \cdot \frac{1}{6} = \frac{1}{3}$$

//

Problem 5. (20 pts) Determine if the series $\sum_{n=1}^{\infty} ne^{-n}$ is convergent or divergent. Justify your answer by referring to a theorem from the course.

Since $a_n = ne^{-n} \geq 0$ for all n.

We may apply integral test on

$f(x) = xe^{-x}$ continuous (Note that $f(x)$ is decreasing)

$$\int_1^\infty xe^{-x}\,dx = \left(-xe^{-x} - e^{-x}\right)\Big|_1^\infty$$

$$= \lim_{t\to\infty}\left(-te^{-t} - e^{-t}\right) - \left(-e^{-1} - e^{-1}\right)$$

$$= (0-0) - \left(-\frac{2}{e}\right) = \frac{2}{e} < \infty$$

$\therefore$ Series converges

Note

$$-\lim_{t\to\infty} te^{-t}$$

$$= -\lim_{t\to\infty} \frac{t}{e^t}$$

$$= -\lim_{t\to\infty} \frac{1}{e^t} = 0$$

L'Hospital

$$f'(x) = e^{-x} - xe^{-x}$$

$$= e^{-x}(1-x)$$

≤ 0 for all $x \geq 1$

Problem 6. (25 pts) Let $a_n = \dfrac{n!}{(2n)!}$. Determine if the sequence a_n is convergent, and if it is, find its limit.

obviously $0 \le a_n$, for all n.

Also note that,

$$a_n = \frac{n!}{(2n)!} = \frac{\cancel{n!}}{\cancel{n!}(n+1)\cdots(2n)}$$

$$\le \frac{1}{n+1} \quad \left(\text{since } n+2, n+3, \ldots, 2n \ge 1\right)$$

$$\therefore \quad 0 \le a_n \le \frac{1}{n+1} \xrightarrow{n\to\infty} 0$$

By squeeze theorem. $a_n \to 0$ //