MATH 31B LECTURE 1 AND 3
PRACTICE MIDTERM

Problem 1. (Multiple choice, 10 pts) Evaluate the improper integral

$$\int_{-1}^{1} \frac{1}{x} dx$$

Indicate your answer in the box below:

(a) 1; (b) 0; (c) -1; (d) integral diverges; (e) None of the above.

Problem 2. (Multiple choice, 10 pts) Find the length of the curve $y = 2\cosh(x)$, $0 \leq x \leq 1$.

(a) $\sinh(1)$; (b) e; (c) $2\sinh(1)\pi$; (d) $2e$; (e) None of the above.

Problem 3. (Multiple choice, 10 pts) Let the sequence $\{a_n\}$ be defined by: $a_1 = 1$, $a_2 = 0$, $a_{n+1} = \frac{1}{2}(\sin(a_n) + \sin(a_{n-1}))$. Assume that it is known that a_n converges to a finite limit L as $n \to \infty$. Determine L. Indicate your answer in the box below:

(a) 0; (b) $\sin(1)$; (c) π; (d) cannot be determined; (e) None of the above.

Problem 4. (25 pts) Consider the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

(a) Determine if the series is convergent;
(b) If the series is convergent, find its limit.

Problem 5. (25 pts) Determine if the series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ is convergent or divergent.

Problem 6. (20 pts) Let $f(x) = \sin(x)$. Find an n so that the trapezoidal rule approximates the integral $\int_0^1 \sin(x)dx$ with an error of no more than $1/120$.

Problem 7. (20 pts) Determine if the series $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ is convergent or divergent.

Problem 8. (20 pts) Determine if the series $\sum_{n=2}^{\infty} \frac{1}{n \log n}$ is convergent or divergent.