Math 31B Homework 8 Solutions

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Section 12.4

3. CONVERGENT

$$\sum_{n=1}^{\infty} \frac{1}{n^2+n+1} \leq \sum_{n=1}^{\infty} \frac{1}{n^2}$$

The right hand sum converges by p-series ($p = 2 > 1$), so the original series must also converge by the comparison test.

5. CONVERGENT

$$\sum_{n=1}^{\infty} \frac{5}{2+3^n} = 5\sum_{n=1}^{\infty} \frac{1}{2+3^n} \leq 5\sum_{n=1}^{\infty} \frac{1}{3^n} = 5\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n$$

The rightmost sum converges because it is a geometric series with $|r| = \left|\frac{1}{3}\right| < 1$. Therefore the original series must converge as well.

7. DIVERGENT

$$\sum_{n=1}^{\infty} \frac{n+1}{n^2} \geq \sum_{n=1}^{\infty} \frac{n}{n^2} = \sum_{n=1}^{\infty} \frac{1}{n}$$

The rightmost sum diverges by p-series ($p = 1$), so the original series must also diverge.

9. CONVERGENT

$$\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2+1} \leq \sum_{n=1}^{\infty} \frac{1}{n^2+1} \leq \sum_{n=1}^{\infty} \frac{1}{n^2}$$

The rightmost sum converges by p-series, so the original sum must also converge.

Section 12.5

5. CONVERGENT

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}; \ \lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$$

By alternating series test, this series converges.

7. DIVERGENT

$$\sum_{n=1}^{\infty}(-1)^n\frac{3n-1}{2n+1};\ \lim_{n\to\infty}\frac{3n-1}{2n+1}=\frac{3}{2}$$

In fact, $\lim_{n\to\infty}(-1)^n\frac{3n-1}{2n+1}$ does not converge, so by the divergence test, this series diverges.

9. CONVERGENT

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{4n^2+1};\ \lim_{n\to\infty}\frac{1}{4n^2+1}=0$$

By alternating series test, this series converges.

23.

$$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2},\ |\text{error}|<.01$$

We see that the sum converges by the alternating series test: $\lim_{n\to\infty}\frac{1}{n^2}=0$. For a convergent alternating series, $|\text{error}|\leq|b_n|$ for s_n, so we need to find the smallest n for which $|b_n|<.01$.

$$|b_n|=\left|\frac{(-1)^{n-1}}{n^2}\right|=\frac{1}{n^2}<.01$$

$$\Rightarrow 100<n^2\Rightarrow n>10$$

Therefore, we will need 11 terms.

25.

$$\sum_{n=1}^{\infty}\frac{(-2)^n}{n!},\ |\text{error}|<.01$$

We see that the sum converges by the alternating series test: $\lim_{n\to\infty}\frac{2^n}{n!}=0$[1]. We now need to find the smallest n for which $|b_n|<.01$.

$$\left|\frac{(-2)^n}{n!}\right|=\frac{2}{n!}<.01$$

$$\Rightarrow\frac{n!}{2^n}>100$$

Computing $\frac{n!}{2^n}$ for increasing values of n, we see that when $n=7$, $\frac{7!}{2^7}=\frac{5040}{128}=39.375$ and when $n=8$, $\frac{8!}{2^8}=\frac{40320}{256}=157.5$. Therefore, we will need 8 terms.

[1] To see this, we use the squeeze theorem:

$$0\leq\frac{2^n}{n!}\leq\frac{2^n}{n\cdot2^{n-2}}\text{ for large n}$$

Now,

$$\lim_{n\to\infty}0=0\ \text{ and }\ \lim_{n\to\infty}\frac{2^n}{n\cdot2^{n-2}}=\lim_{n\to\infty}\frac{4}{n}=0.$$

Therefore, $\lim_{n\to\infty}\frac{2^n}{n!}=0$