Math 31B Homework 1 Solutions

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Please note that all problems with graphs will be separate.

Section 7.2*

13.

$$f(x) = \sqrt{x}\ln x;\ f'(x) = \frac{\sqrt{x}}{x} + \frac{1}{2}x^{-\frac{1}{2}}\ln x = x^{-\frac{1}{2}}(1 + \frac{1}{2}\ln x)$$

14.

$$f(x) = \ln(x^2 + 10);\ f'(x) = \frac{2x}{x^2 + 10}$$

15.

$$f(\theta) = \ln(\cos\theta);\ \frac{df}{d\theta} = \frac{-\sin\theta}{\cos\theta} = -\tan\theta$$

16.

$$f(x) = \cos(\ln x);\ f'(x) = -\sin(\ln x)\frac{1}{x}$$

17.

$$f(x) = \sqrt[5]{\ln x} = (\ln x)^{\frac{1}{5}};\ f'(x) = \frac{1}{5}(\ln x)^{-\frac{4}{5}}\frac{1}{x}$$

18.

$$f(x) = \ln\sqrt[5]{x} = \frac{1}{5}\ln x;\ f'(x) = \frac{1}{5x}$$

19.

$$g(x) = \ln\left(\frac{a-x}{a+x}\right) = \ln(a-x) - \ln(a+x);$$

$$g'(x) = \frac{-1}{a-x} - \frac{1}{a+x}$$

20.

$$h(x) = \ln(x + \sqrt{x^2-1});$$

$$h'(x) = \frac{1 + \frac{1}{2}(x^2-1)^{-\frac{1}{2}}2x}{x + \sqrt{x^2-1}} = \frac{1 + x(x^2-1)^{-\frac{1}{2}}}{x + \sqrt{x^2-1}} = \frac{\sqrt{x^2-1} + x}{x\sqrt{x^2-1} + x^2 - 1}$$

21.

$$f(u) = \frac{\ln u}{1 + \ln(2u)};$$

$$\frac{df}{du} = \frac{[1+\ln(2u)]\frac{1}{u} - \ln(u)\frac{1}{2u}2}{[1+\ln(2u)]^2} = \frac{1+\ln(2u)-\ln(u)}{u[1+\ln(2u)]^2} = \frac{1+\ln 2}{u[1+\ln(2u)]^2}$$

22.

$$f(t) = \frac{1+\ln t}{1-\ln t};$$

$$\frac{df}{dt} = \frac{(1-\ln t)\frac{1}{t} - (1+\ln t)(-\frac{1}{t})}{(1-\ln t)^2} = \frac{1-\ln t+1+\ln t}{t(1-\ln t)^2} = \frac{2}{t(1-\ln t)^2}$$

55.

$$y = (2x+1)^5(x^4-3)^6;\ \ln y = 5\ln(2x+1) + 6\ln(x^4-3)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{5\cdot 2}{2x+1} + \frac{6\cdot 4x^3}{x^4-3} = \frac{10x^4-30+48x^4+24x^3}{(2x+1)(x^4-3)}$$

$$\frac{dy}{dx} = \frac{58x^4+24x^3-30}{(2x+1)(x^4-3)}(2x+1)^5(x^4-3)^6 = 2(29x^4+12x^3-15)(2x+1)^4(x^4-3)^5$$

56.

$$y = \frac{(x^3+1)^4\sin^2 x}{x^{\frac{1}{3}}};\ \ln y = 4\ln(cx^3+1) + 2\ln(\sin x) - \frac{1}{3}\ln x$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{12x^2}{x^3+1} + \frac{2\cos x}{\sin x} - \frac{1}{3x}$$

$$\frac{dy}{dx} = \frac{(x^3+1)^4\sin^2 x}{x^{\frac{1}{3}}}\left(\frac{12x^2}{x^3+1} + \frac{2\cos x}{\sin x} - \frac{1}{3x}\right)$$

57.

$$y = \frac{\sin^2 x\tan^4 x}{(x^2+1)^2};\ \ln y = 2\ln(\sin x) + 4\ln(tanx) - 2\ln(x^2+1)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2\cos x}{\sin x} + \frac{4\sec^2 x}{\tan x} - \frac{2\cdot 2x}{x^2+1} = 2\cot x + 4\sec x\csc x - \frac{4x}{x^2+1}$$

$$\frac{dy}{dx} = \frac{\sin^2 x\tan^4 x}{(x^2+1)^2}\left(2\cot x + 4\sec x\csc x - \frac{4x}{x^2+1}\right)$$

Section 7.3*

51. $f(x) = e^{2x};\ f^{(n)}(x) = 2^n e^{2x}$

63. See answers on p. A83 of Stewart

64. A: Domain: all x

B: Intercepts: $(0,0)$

C: Symmetry: none

D: Asymptotes: horizontal: y=0 (as $x \to -\infty$); vertical: none

E: increasing: $x > -\ln 2$; decreasing: $x < -\ln 2$

F: local max: none; local min: $(-\ln 2, -\frac{1}{4})$

G: concave up: $x > -\ln 4$; concave down: $x < -\ln 4$;
inflection points: $(-\ln 4, -\frac{3}{16})$

H: see graphs

65. See answers on p. A83 of Stewart

Section 7.3

61. $y = \ln(x+3)$;

$$\text{inverse: } x = \ln(y+3) \Rightarrow e^x = y+3 \Rightarrow y = e^x + 3$$

62. $y = 2^{10^x}$;

$$\text{inverse: } x = 2^{(10^y)} \Rightarrow \log_2 x = 10^y \Rightarrow y = \log_{10}(\log_2 x)$$

63. $f(x) = e^{x^3}$;

$$f^{-1}(x) = (\ln x)^{\frac{1}{3}}$$

64. $y = (\ln x)^2, x \geq 1$;

$$\text{inverse: } x = (\ln y)^2, y \geq 1 \Rightarrow \sqrt{x} = \ln y \Rightarrow y = e^{\sqrt{x}}$$

65. $y = \frac{10^x}{10^x+1}$;

$$\text{inverse: } x = \frac{10^y}{10^y+1} \Rightarrow x10^y + x = 10^y \Rightarrow x = 10^y(1-x)$$

$$\Rightarrow 10^y = \frac{x}{1-x} \Rightarrow y = \log_{10}\frac{x}{1-x}$$

66. $y = \frac{1+e^x}{1-e^x}$;

$$\text{inverse: } x = \frac{1+e^y}{1-e^y} \Rightarrow x - xe^y = 1 + e^y \Rightarrow \frac{x-1}{x+1} = e^y \Rightarrow y = \ln\left(\frac{x-1}{x+1}\right)$$

Section 7.4

5.

$$f(x) = \log_2(1-3x);\ f'(x) = \frac{-3}{\ln 2(1-3x)}$$

65.

$$\int_2^4 \frac{3}{x}dx = 3\ln x\big|_2^4 = 3\ln 4 - 3\ln 2 = 3\ln\frac{4}{2} = 3\ln 2$$

66.

$$\int_1^2 \frac{4+u^2}{u^3}du = \int_1^2 \left(4u^{-3} + \frac{1}{u}\right)du = (-2u^{-2} + \ln u)\Big|_1^2$$

$$= \frac{-2}{4} + \frac{2}{1} + \ln 2 - 0 = \frac{3}{2} + \ln 2$$

67.

$$\int_1^2 \frac{dt}{8-3t} = -\frac{1}{3}\ln|8-3t|\Big|_1^2 = -\frac{1}{3}(\ln 2 - \ln 5) = \frac{1}{3}\ln\frac{5}{2}$$

68.

$$\int_4^9 \left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)^2 dx = \int_4^9 \left(x + 2 + \frac{1}{x}\right) dx = \left(\frac{x^2}{2} + 2x + \ln x\right)\Big|_4^9$$

$$= \frac{81}{2} + 18 + \ln 9 - 8 - 8 - \ln 4 = \frac{85}{2} + 2\ln\frac{3}{2}$$

69.

$$\int_1^e \frac{x^2 + x + 1}{x} dx = \int_1^e \left(x + 1 + \frac{1}{x}\right) dx = \left(\frac{x^2}{2} + x + \ln x\right)\Big|_1^e$$

$$= \frac{e^2}{2} + e + 1 - \frac{1}{2} - 1 - 0 = \frac{e^2}{2} + e - \frac{1}{2}$$

70. Using substitution $u = \ln x$:

$$\int_e^6 \frac{dx}{x \ln x} = \int_1^{\ln 6} \frac{du}{u} = \ln u\Big|_1^{\ln 6} = \ln(\ln 6) - 0 = \ln(\ln 6)$$

Section 7.4*

26. $y = 10^{\tan\theta}$;

$$\frac{dy}{d\theta} = \ln 10 \cdot 10^{\tan\theta} \cdot \sec^2\theta$$

27. $f(u) = (2^u + 2^{-u})^{10}$;

$$\frac{df}{du} = 10(2^u + 2^{-u})^9(\ln 2 \cdot 2^u - \ln 2 \cdot 2^{-u}) = 10\ln 2(2^u + 2^{-u})^9(2^u - 2^{-u})$$

28. $y = 2^{3^{x^2}}$;

$$\frac{dy}{dx} = \ln 2(2^{3^{x^2}})\ln 3(3^{x^2})2x$$

29. $f(x) = \log_3(x^2 - 4)$;

$$f'(x) = \frac{1}{\ln 3} \cdot \frac{2x}{x^2 - 4}$$

30.

$$f(x) = \log_{10}\left(\frac{x}{x-1}\right) = \frac{1}{\ln 10}\ln\left(\frac{x}{x-1}\right) = \frac{1}{\ln 10}\left(\ln x - \ln(x-1)\right);$$

$$f'(x) = \frac{1}{\ln 10}\left(\frac{1}{x} - \frac{1}{x-1}\right)$$

31. $y = x^x$;

$$\ln y = x\ln x;\ \frac{1}{y}\frac{dy}{dx} = \frac{x}{x} + \ln x$$

$$\frac{dy}{dx} = x^x(1 + \ln x)$$

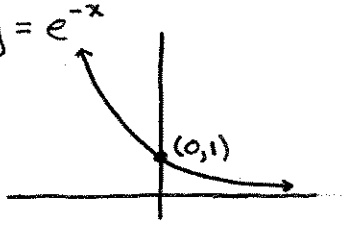
32. $y = x^{\frac{1}{x}}$;

$$\ln y = \frac{1}{x}\ln x;\ \frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2} - \frac{1}{x^2}\ln x$$

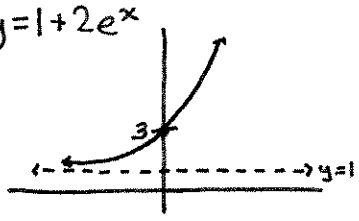
$$\frac{dy}{dx} = x^{\frac{1}{x}}\left(\frac{1}{x^2}\right)(1 - \ln x) = x^{\frac{1}{x}-2}(1 - \ln x)$$

7.3*

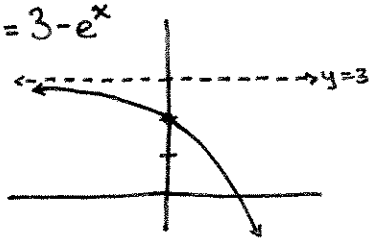
19. $y = e^{-x}$



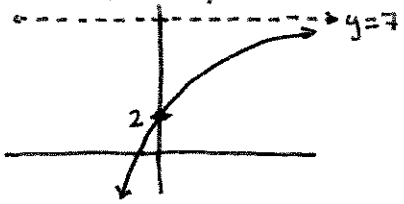
20. $y = 1 + 2e^{x}$



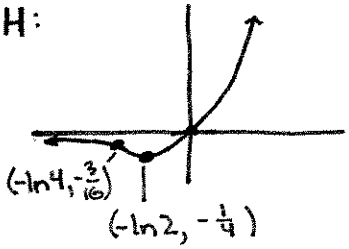
21. $y = 3 - e^{x}$

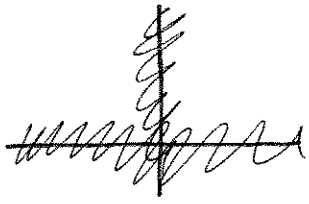


22. $y = 2 + 5(1 - e^{-x}) = 7 - 5e^{-x}$



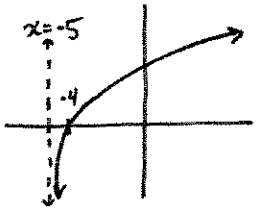
64. H:



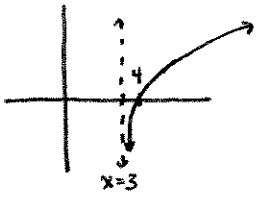


7.3

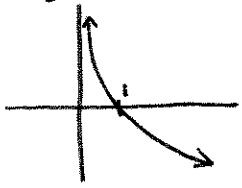
23. $y = \log_{10}(x+5)$



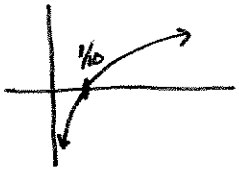
24. $y = \log_2(x-3)$



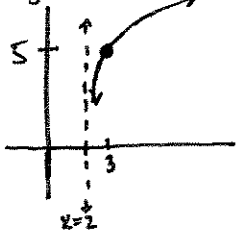
25. $y = -\ln x$



26. $\ln(10x) = y$



27. $y = 5 + \ln(x-2)$



28. $y = \ln|x|$