MATH 31B SECTIONS 1 AND 3
PRACTICE FINAL EXAM.

Please note: Show your work. Except on multiple-choice problems, correct answers not accompanied by sufficent explanations will receive little or no credit. Please call one of the proctors if you have any questions about a problem. No calculators, computers, PDAs, cell phones, or other devices will be permitted.

#1	#2	#3	#4	#5	#6	
#7	#8	#9	#10	#11	#12	Total

SID:_____ TA: _____ Section(circle): Tuesday Thursday

Name:_____

Problem 1. Let $f(x) = \sum_{n=0}^{\infty} \frac{3n}{2n+5} x^n$. Find the values of $f(0)$, $f'(0)$ and $f''(0)$.

Problem 2. Let

$$f(x) = \begin{cases} \sqrt{x} & x \geq 0 \\ -\sqrt{-x} & x < 0. \end{cases}$$

Show that the function f is one-to-one on $(-\infty, \infty)$ and find its inverse function.

Problem 3. Let $f(x) = \tan^{-1} x$.

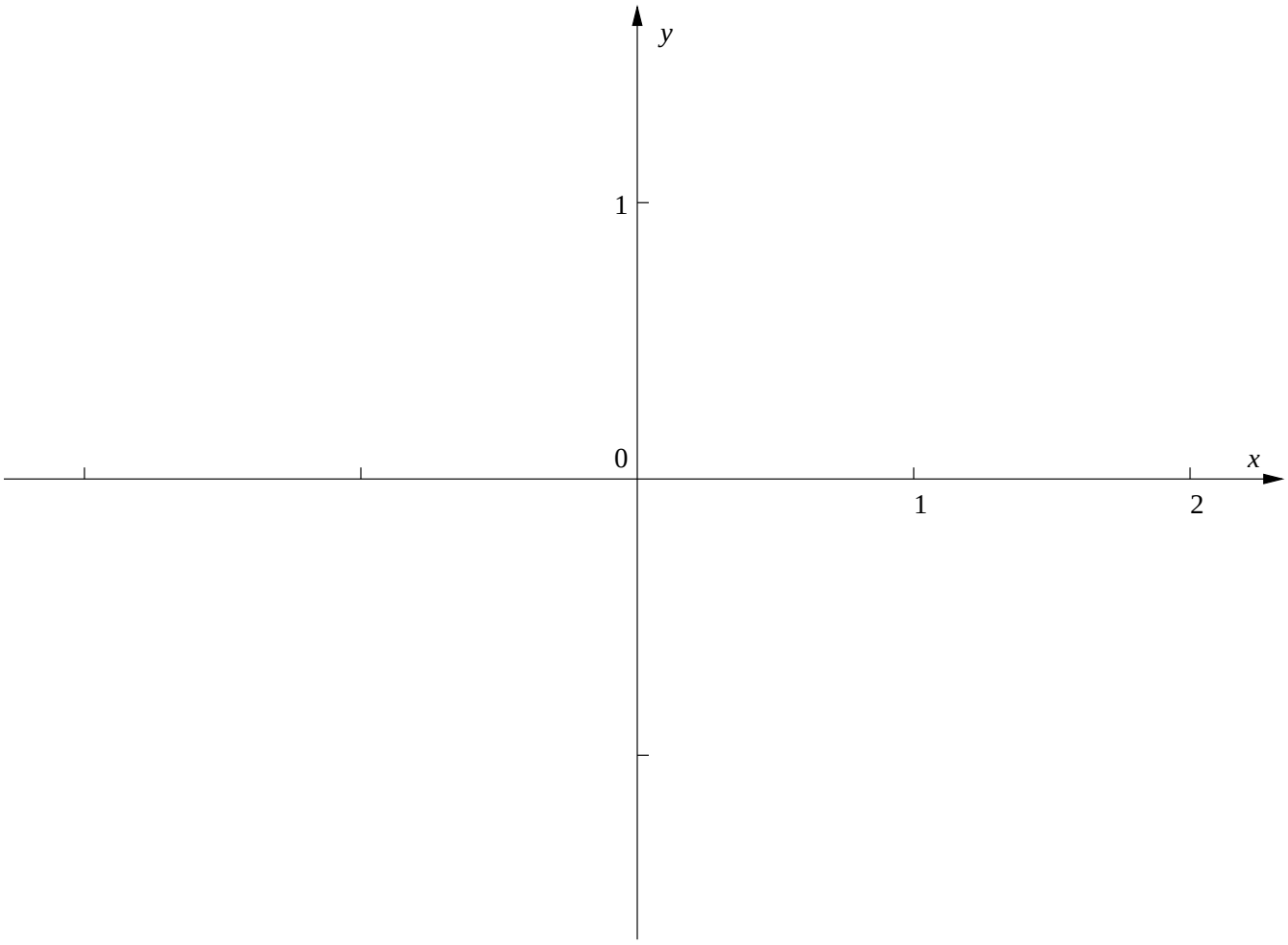
(a)Find a power series representation for f around 0. (*Hint:* represent $\tan^{-1} x$ as an integral).

(b) Show the series conveges to $f(x)$ if $-1 < x < 1$.

Problem 4. Is the improper integral $\int_0^\infty x^2 e^{-x} dx$ convergent or divergent? Explain.

Problem 5. Evaluate the integral $\int \ln x dx$.

Problem 6. Graph the function $x \ln x$ on the interval $(0, +\infty)$. Indicate minima, maxima, convexity and inflection points, as well as the limits of the function as $x \to 0$ and $x \to +\infty$. Use the coordinate axes below:



(use the next page if you need extra space)

(turn page for more problems)

Problem 7. (a) Assume $\lim_{n\to\infty} A_n = A$ and $\lim_{n\to\infty} B_n = B$. State a theorem about $\lim_{n\to\infty} A_{n+1}$ and another theorem about $\lim_{n\to\infty}(A_n + B_n)$ and $\lim_{n\to\infty}(A_n - B_n)$.

(b) Use the theorems from (a) to prove that if the infinite series $\sum_{n=1}^{\infty} a_n$ converges, then the sequence of terms a_n has limit 0.

Problem 8. Use a trigonometric substitution to evaluate the integral $\int_0^1 \frac{r^2}{\sqrt{1-r^2}} dr$.

Problem 9. Let $f(x) = \dfrac{x+1}{(x-1)(x-2)(x-3)}$. Use partial fractions to evaluate the integral $\displaystyle\int f(x)\,dx$.

Problem 10. Determine the radius and interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(x-1)^n}{2^n+3}$. Be sure to check the convergence at the endpoints of the interval!

Problem 11. Let $S_n = \sum_{k=1}^{n} (-1)^k \frac{1}{\sqrt{k}}$.

By a theorem, the limit $S = \lim_{n\to\infty} S_n$ exists. Find a number n such that

$$|s_n - S| < 1/(25).$$

Problem 12. How many terms do you need to keep in the Taylor series expansion for e^x around zero to approximate the number e to 10 decimal places? You may use that $e < 3$.