

MATH 31B/2 PRACTICE FINAL EXAM.

Please note: the aim of this practice final is to give you several problems on the material not covered by the first two midterms and practice midterms. ***You should therefore treat the first two midterms and practice midterms as a part of the practice final.***

- (1) Let $f(x) = \sin x$. Find n so that Taylor's polynomial of degree n around 0 approximates $\sin(1)$ to within 10^{-2} . Justify your answer.
- (2) Let

$$f(x) = \frac{2x + 4}{x^3 - 1}.$$

Express $f(x)$ as a sum of terms using partial fractions. Use this to evaluate $\int f(x)dx$.

- (3) Find the Taylor series for the function $f(x) = \ln|x - 1|$. Determine its radius and interval of convergence.
- (4) Find the power series representation of the function $\frac{1}{(1 - x)^2}$. Determine its radius and interval of convergence.
- (5) Find the limit of the sequence $a_n = n^{1/n}$.
- (6) Is the improper integral $\int_0^\infty \sin(e^x)dx$ convergent or divergent? Explain.
- (7) Find the surface area of the surface of revolution obtained by rotating the parabola $y = x^2$, $0 \leq x \leq 1$, about the y -axis.
- (8) Use the arclength formula to find the length of the circle of radius 1.
- (9) Determine whether the following series are absolutely convergent, conditionally convergent, or divergent: (a) $\sum (-1)^n \frac{1}{n + \sin(n)}$; (b) $\sum (-1)^n \frac{1}{n^2 + \sin(n)}$; (c) $\sum e^{-n}$.