MATH 132
PRACTICE MIDTERM

Problem 1. State the book's definition of:
(a) The functions $\sin z$, $\cos z$, $\mathrm{Log}(z)$ (principal branch), $\sqrt{z}$ (Principal branch), e^z
(b) Continuity of a function at z_0
(c) Analytic function

Problem 2. Show that if a function $f(z)$ is analytic, it is continuous. Prove that the function $f(z) = \cos z$ is analytic (use Cauchy-Riemann equations). Conclude that $f(z)$ is continuous.

Problem 3. For each of the following functions, find the image of the indicated regions: $f(z) = \frac{1+z}{1-z}$, $g(z) = \sqrt[4]{z}$(principal branch), $h(z) = \mathrm{Arg}(z)$:
(a) The unit disk $|z| \leq 1$; (b) the upper half-plane $\mathrm{Im} z > 0$; (c) the real axis $\mathrm{Im} z = 0$.

Problem 4. Let $u(x + iy) = \log |x + iy|$. Find a harmonic conjugate to u.

Problem 5. Let D be a region bounded by a simple closed curve γ. Express the integral $\int_\gamma x dy - y dx$ in terms of the area of D.

Problem 6. Prove the identity $\cos(z+w) = \cos z \cos w - \sin z \sin w$ using the definition of $\cos z$ in terms of complex exponentials and the properties of complex exponentials.

Problem 7. Let $f(z) = z(z - 1)$. Show from the definition that $f(z)$ is continuous. (Hint: prove an estimate of the form $|f(z) - f(z_0)| < C|z - z_0|$ for z near z_0).