PRACTICE PROBLEMS FOR THE FINAL.

Problem 1. Let f, g be continuous functions of period 2π, with Fourier coefficients $c_k$and d_k. Prove that

$$\langle f, g\rangle_{L^2} = \sum_k c_k \bar{d}_k.$$

Note that this gives Parseval's relation in the case that $f = g$.

Problem 2. Prove the Riemann-Lebesgue lemma: if f is piece-wise continuous on $[a, b]$, then

$$\lim_{n\to\infty} \int_a^b f(t)\exp(int)dt = 0.$$

Problem 3. Let f be a complex-valued 2π periodic function, and consider the Fourier coefficients

$$c_k = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} f(t)\exp(-ikt)dt.$$

Find a condition on c_k that holds if and only if f is real-valued.

Problem 4. Find the radius of convergence of the series

$$\sum_{j=0}^{\infty} (j+1)(j+2)x^j.$$

Find the function to which this power series converges.