

Round 2 Rules

Round 2 will be a Jeopardy! style contest between the top three scorers of round 1. The rules are the same as those in Jeopardy! with a few exceptions. Firstly, there is no Daily Double, and there is no Double Jeopardy (but there is Final Jeopardy). Secondly, contestants may buzz-in as soon as the question appears on screen, but they only have 2 seconds to begin their response once their name is called by Sam. Recall that answering incorrectly or failing to start answering in time means you *lose* the value of the question. Finally, if a contestant loses points, the remaining contestants will still have the remaining time to try and solve the question and buzz in.

Questions below the 500 point level will be by default 1 minute, and the 500 level will be 3 minute. Sam will grant extensions to all participants as needed. Questions are still computational in nature, although you may have to compute general real numbers, functions, groups, etc.

The questions fall in to six categories, chosen based on the most heavily enrolled and commonly offered upper-division math courses.

1. **ALGEBRA: 115A, 110A**, 115B, 110B
2. **ANALYSIS: 131A, 132**, 131B, 121
3. **PROBABILITY, COMBINATORICS, & STATISTICS: 170E, 170S**, 170A, 170B, 171
4. **DIFFERENTIAL EQUATIONS: 151A, 151B**, 142, 164
5. **NUMERICAL METHODS: 134, 135**, 120A, 136
6. **MISCELLANEOUS:** Any math course or course listed in a math major is possible

Bolded classes are considered “core” classes and generally have more questions assigned. Honors variants are included when applicable.

Question point value is based on a combination of how specialized is the knowledge, how likely are mistakes, and how much time it will take to compute. Generally, the 100-point level questions are quick and should be accessible to people with even a basic idea of what goes on in the relevant courses. The 500-point level questions almost always require very specialized knowledge or lengthy, error-prone computations.

No justification/proof is required for answers, although in the event of a dispute, some justification may be needed. Please be nice to me I don't actually know half of this.

Final Jeopardy will begin once either all 30 questions are attempted or the time reaches 6:55.

The Board

<https://jeopardylabs.com/play/are-you-smarter-than-an-undergrad-round-2-2>

Final Jeopardy

The category is “bases”.

Let V be the real vector space of smooth functions $f : [-2, 2] \rightarrow \mathbb{R}$, and let W be the subspace generated by $\{1, x, x^2\}$. We can make W into an inner product space via the inner product $(f(x), g(x)) := \int_{-2}^2 f(x)g(x)dx$. Find an orthonormal basis for W that includes the vector $f_1(x) = 1/2$.

Final Jeopardy Answer

The most common answer will be the one you get if you apply Gram-Schmidt to the ordered basis $\{1, x, x^2\}$, which gives you $f_1(x) = 1/2$, $f_2(x) = \sqrt{3}x/4$, $f_3(x) = \frac{3\sqrt{5}}{16}(x^2 - 4/3)$. You can use Wolfram Alpha to check the required inner product relations.