

## Workshop 1

1. Let  $V$  be a finite dimensional complex vector space and let  $G$  be a finite subgroup of  $GL(V)$ . Prove that

$$|G|^{-1} \sum_{g \in G} \text{Trace}(g) = \dim(V^G)$$

where  $V^G$  is the subspace of vectors fixed by  $G$ .

[Hint. Consider the linear transformation  $|G|^{-1} \sum_{g \in G} g$ .]

2. Let  $M_i, i \in I$ , and  $N$  be  $R$ -modules. Show

a.  $\text{Hom}_R(\prod_I M_i, N) \cong \prod_I \text{Hom}_R(M_i, N)$

b.  $\text{Hom}_R(N, \prod_I M_i) \cong \prod_I \text{Hom}_R(N, M_i)$

3. Show the natural exact sequence of abelian groups

$$0 \rightarrow \mathbf{Z} \rightarrow \mathbf{Q} \rightarrow \mathbf{Q}/\mathbf{Z} \rightarrow 0$$

does not split.

4. Let  $n$  and  $m$  be positive integers. Construct an exact sequence of abelian groups

$$0 \rightarrow \mathbf{Z}/n\mathbf{Z} \rightarrow \mathbf{Z}/nm\mathbf{Z} \rightarrow \mathbf{Z}/m\mathbf{Z} \rightarrow 0.$$

Determine when this sequence splits.

5. Let  $P$  be an  $R$ -module. Show that  $P$  is  $R$ -projective if and only if there exist a subset  $\{x_i\}_{i \in I} \subset P$  and  $\phi_i \in \text{Hom}_R(P, R)$ ,  $i \in I$ , such that for all  $x \in P$ , we have  $\phi_i(x) = 0$  for almost all  $i$  and  $x = \sum \phi_i(x)x_i$ .
6. (Schanuel's Lemma) Suppose we have two exact sequences of  $R$ -modules

$$0 \rightarrow K_i \rightarrow P_i \rightarrow M \rightarrow 0$$

with  $P_i$  projective for  $i = 1, 2$ . Prove that  $P_1 \amalg K_2 \cong P_2 \amalg K_1$ .