

# GEOMETRIC REPRESENTATION THEORY, MATH 216A

RAPHAËL ROUQUIER

*Spring 2024*

MWF 10-10:50am, MS 6229

This course is an introduction to geometric representation theory. This is a branch of representation theory that uses geometric methods to construct representations or categories of representations of Lie algebras and related structures.

The first part will be devoted to localization theory. Its original incarnation is the construction of enveloping algebras of complex semi-simple Lie algebras as global sections of differential operators on flag varieties. Via relative Hodge theory, this provides character formulas for simple representations in terms of Kazhdan-Lusztig polynomials.

The second part will introduce derived algebraic geometry and  $\infty$ -category methods. The main topic will be the Hecke category and character sheaves. This is related to the geometric Langlands program and to representations of finite groups of Lie type.

Topics to be discussed:

- Kazhdan-Lusztig polynomials and Lie algebra representations
- Flag varieties
- $\mathcal{D}$ -modules
- Derived algebraic geometry
- Hecke category, character sheaves

Prerequisites: Basic representation theory, Lie theory, algebraic geometry.

**Office Hours:** By appointment.

## Grading

The course assessment will be based on a 20mn presentation of an assigned project. The presentations will be held during the period June 3–7. A list of possible projects will be provided on Wednesday April 24 and a preferred choice, as well as a second and third choice, will need

to be sent to me by email by Friday April 26. You may also suggest your own topic, which I will need to approve.

An abstract of what you plan to do for your presentation will be due on Wednesday May 15.

### References

P. Achar, *Perverse Sheaves and Applications to Representation Theory*, AMS, 2021.

A. Borel, *Linear Algebraic Groups*, 2nd edition, Springer Verlag, 1997.

R. Hotta, K. Takeuchi and T. Tanisaki, *D-modules, Perverse Sheaves, and Representation Theory*, Birkhäuser, 2008.

M. Kashiwara, *Representation theory and  $\mathcal{D}$ -modules on flag varieties*, *Astérisque* **173–174** (1989), pp. 55–109.