A note on the opening of a futures market and its effect on the spot price

Esther Bruegger and Roberto H. Schonmann *

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Abstract

In a standard finance economy model we show that under fairly general conditions, opening a futures market has the same effect on an asset's spot price as releasing short sale constraints: If people have heterogenous expectations on how the price develops in the future, the spot price decreases when the futures market opens.

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*Address: UCLA Mathematics Department, Box 951555, Los Angeles, CA 90095-1555. Email addresses: bruegger@math.ucla.edu, rhs@math.ucla.edu. The authors thank the NSF for financial support (grant number DMS 0300672).

1 Introduction

In the spring of 2006 MACRO Securities Research and the Chicago Mercantile Exchange launched futures contracts based on the Case Shiller Indexes which mirror the housing prices for ten major US cities.[1] The futures contracts allow for investments in the real estate markets without actual purchases of housing and therefore eliminate significant up-front transaction costs of real estate investments.

In this note we are concerned with the effect of such an event on the spot market price of the underlying asset (housing in our example). In our analysis we focus on the fact that people make different forecasts of the price of an asset in the foreseeable future. Assuming heterogenous expectations of price developments, Miller (1977), Jarrow (1980), Diamond and Verrecchia (1987), or Hietala et al. (2000) among others have discussed the fact that short sale restrictions in asset markets prevent the price from fully reflecting the information possessed by all market participants. Moreover, under certain conditions, short sale restrictions increase the spot price of an asset.[2]

This note uses a simple, but very general, finance economy model to argue that the opening of a futures market has the same effect on the spot price as the lifting of short sale restrictions. Our main conclusion is that the spot price should drop when the futures market opens.

The analysis applies to markets for which ownership of an asset or commodity results in a net income proportional to the amount owned. For concreteness, we will refer to the housing market in this note, but emphasize here that nothing special about housing, as opposed to other assets, is being used. After the presentation of the model, we discuss some general features of asset markets that affect the conclusions drawn from our model.

[1] See macromarkets.com.

[2] In the housing market short sale constraints exist naturally. The reason is that nobody can sell a house he or she does not own and that real estate investment trusts, of which shares can be short sold, account for a small fraction of the real estate market only.

2 The model

2.1 The finance economy

We apply the general finance economy model (e.g., Lengwiler, 2004) with two time periods, denoted by 0 and 1, to the housing market. The asset of the economy is a standardized unit of housing which we will call a house for simplicity.[3] In period 0, each agent i has the initial endowment W_i and makes two decisions: First, he decides on the number K_i^R of houses to buy at the spot market price p_0 (R stands for real asset). Second, he decides to save m_i at the risk free interest rate which we set to zero for simplicity. We assume that each house pays the rent r to its owner in period 1 and that there are no borrowing constraints, so that m_i can take any real value.[4]

An agent's decisions depend on his expectation of what the value of a house will be in period 1. We denote the price of a house in period 1 by the random variable $\tilde{p}_1$. Note that we explicitly allow for heterogenous expectations of $\tilde{p}_1$. The decision problem of agent i takes the following form:

$$\max_{m_i, K_i^R \geq 0} \left\{ u_{0,i}(W_i - m_i - p_0 K_i^R) + E_i[u_{1,i}(m_i + \tilde{p}_1 K_i^R + r K_i^R)] \right\}, \tag{1}$$

where the utility functions, $u_{0,i}$ and $u_{1,i}$, are supposed to satisfy standard conditions reflecting risk aversion, i.e., they are increasing and strictly concave. The constraint $K_i^R \geq 0$ can be seen as the impossibility of short sales in the housing market.

We define savings as $S_i = m_i + p_0 K_i^R$. Since there are no borrowing constraints agents can choose their savings S_i freely. So agent i's maximization problem can be written as

$$\max_{S_i, K_i^R \geq 0} \left\{ u_{0,i}(W_i - S_i) + E_i[u_{1,i}(S_i + (\tilde{p}_1 - p_0 + r) K_i^R)] \right\}. \tag{2}$$

[3] We discuss the effects of the discontinuous nature that is inherent to the purchase of housing in Section 3.

[4] Note that a zero interest rate implies that the endowment can be shifted costlessly, it does therefore not matter to which period we assign it.

In the following, we use $K_i^R(p_0)$ to denote agent i's direct demand function for houses, which is the K_i^R occurring in the solution of his maximization problem (2).

Note that only agents with $E_i[\tilde{p}_1] > p_0 + r$ will demand a $K_i^R(p_0) > 0$ (see Gollier, 2004, p. 54, Proposition 6). Agents with $E_i[\tilde{p}_1] \leq p_0 + r$ choose $K_i^R(p_0) = 0$. We suppose that agent i's demand for houses $K_i^R(p_0)$ is continuous and strictly decreasing in p_0 when it is positive. A sufficient condition for this is that agents have constant absolute risk aversion (CARA) utility functions (see Appendix A).

Let H be the total number of houses on the market. Under the conditions above, there is a unique equilibrium price p_0^* which equalizes demand and supply for housing, i.e., satisfies

$$\sum_i K_i^R(p_0^*) = H \,. \tag{3}$$

See Figure 1.

2.2 Opening the futures market

We now allow for a futures market of the standardized house.[5] The number of houses agent i buys on the futures market for the futures price F is denoted by K_i^F. If $K_i^F >$ ($<$) 0, agent i takes a long (short) position, i.e., makes (loses) money when the price for a house in period 1 turns out to be higher than F. His decision problem now takes the following form:

$$\begin{aligned} &\max_{m_i,\, K_i^R \geq 0,\, K_i^F} \quad \Big\{ \; u_{0,i}(W_i - m_i - p_0 K_i^R) \\ &\qquad\qquad + E_i[u_{1,i}(m_i + \tilde{p}_1 K_i^R + r K_i^R + (\tilde{p}_1 - F) K_i^F)] \Big\} \qquad (4) \\ &= \\ &\max_{S_i,\, K_i^R \geq 0,\, K_i^F} \quad \Big\{ \; u_0(W_i - S_i) + E_i[u_{1,i}(S_i + (\tilde{p}_1 - p_0 + r)(K_i^R + K_i^F))] \Big\} . \qquad (5) \end{aligned}$$

Equality between (4) and (5) follows from a no-arbitrage condition implying

$$F = p_0 - r \tag{6}$$

[5]This corresponds to a futures market on the housing price index as described in the Introduction.

(see e.g. Hull, 2005, Equation (5.2))[6] and from the definition of the variable $S_i = m_i + p_0 K_i^R$, which as before we can interpret as savings.

We see in (5), that K_i^R and K_i^F only appear as a sum in the optimization problem. Consequently, K_i^R and K_i^F are perfect substitutes in this model.[7] Consequently, we can replace $K_i^R + K_i^F$ with the variable K_i, which means that solving (4) is equivalent to solving[8]

$$\max_{S_i, K_i} \left\{ u_{0,i}(W_i - S_i) + E_i \left[u_{1,i}(S_i + (\tilde{p}_1 - p_0 + r)K_i) \right] \right\}. \tag{7}$$

Note that (7) is the same as maximization problem (2), except that $K_i \in I\!\!R$, while $K_i^R \in I\!\!R_+$. As in the previous section we denote agent i's direct demand function for houses and positions in the futures market, i.e., the K_i which occurs in the solution of the maximization problem (7), by $K_i(p_0)$.

The solution $K_i(p_0)$ to (7) is assumed to be continuous and strictly decreasing in p_0 (which is satisfied in the CARA case as shown in the Appendix). So the equilibrium price p_0^{**} which satisfies

$$\sum_i K_i(p_0^{**}) \quad = \quad H \tag{8}$$

is unique. The right hand side of (8) is the supply of houses as in the case without the futures market displayed in (3). The reason is that to each long position in a futures market there must correspond a short position, so the positions in the futures market cancel out.

Note that we can write the total demand as

$$\sum_i K_i(p_0) 1\!\!1_{\{K_i(p_0)>0\}} + \sum_i K_i(p_0) 1\!\!1_{\{K_i(p_0)<0\}}.$$

Since the two maximization problems, (2) and (7), are the same for those agents

[6] In the 5th edition, see Equation (3.6). We present a self-contained derivation of (6) in Appendix B.

[7] The reason is that agents value housing only as a source of income, we will discuss this feature of the model in Section 3.

[8] To see this, note that if S_i, K_i^R and K_i^F solve (5), then clearly S_i and $K_i = K_i^R + K_i^F$ solve (7). Conversely, if S_i and K_i solve (7), then S_i, $K_i^R = 0$ and $K_i^F = K_i$ solve (5).

with $E_i[\tilde{p}_1] > p_0 + r$, we have $\sum_i K_i(p_0)\mathbb{1}_{\{K_i(p_0)>0\}} = \sum_i K_i^R(p_0)$. Consequently,

$$\begin{aligned}\sum_i K_i^R(p_0^{**}) &= H - \sum_i K_i(p_0^{**})\mathbb{1}_{\{K_i(p_0^{**})<0\}} \\ &= H + \sum_i |K_i(p_0^{**})|\mathbb{1}_{\{K_i(p_0^{**})<0\}}\,. \end{aligned} \tag{9}$$

From (3) and (9) it is clear that $\sum_i K_i^R(p_0^{**}) \geq \sum_i K_i^R(p_0^*)$, and hence

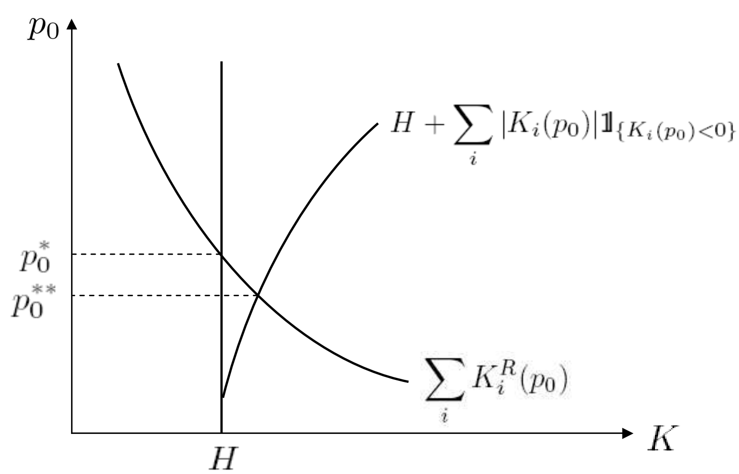
$$p_0^{**} \leq p_0^*\,. \tag{10}$$

If at least one agent i has $E[\tilde{p}_1] < p_0^* + r$, then we argue that

$$p_0^{**} < p_0^*\,.$$

Suppose otherwise; then it follows from (10) that we have $p_0^{**} = p_0^*$ and therefore $E[\tilde{p}_1] < p_0^{**} + r$. Consequently, $K_i(p_0^{**}) < 0$, which in combination with (3) and (9) implies $\sum_i K_i^R(p_0^{**}) > \sum_i K_i^R(p_0^*)$, and hence $p_0^{**} < p_0^*$.

Intuitively, (9) shows that the opening of the futures market has the same effect as the enlargement of the supply of housing. We illustrate this in Figure 1.



The opening of the futures market allows the expectations of agents who are pessimistic about the housing price in period 1 – and who do therefore not buy housing in period 0 – to filter into the market. This happens as these agents take short positions in the futures market and arbitrageurs then act on the spot market causing its price to equilibrate at $p_0 = F + r$, as given by the no-arbitrage condition (6). Specifically, arbitrageurs here are agents who own housing and, as they see $F < p_0 - r$, sell their housing and take a corresponding long position in the futures market. Their riskless profit in period 1 is $p_0 - r - F > 0$ per house sold, the spot price decreases while the futures market increases until the no-arbitrage condition is satisfied.

This effect is similar to the introduction of unconstrained short sales into the spot market. The fact that spot prices decrease when short sales constraints are removed is intuitive. In the setup we study, the opinion of agents who are optimistic about the spot price in period 1 can filter into the spot market as they bid up the prices of houses. The opinion of pessimistic agents however, is prevented from filtering into the market. The creation of a futures market, even though it contains both a short and a long side, does not influence the spot price symmetrically. With the long side it just provides a substitute to owning a house. In the short side however, it removes a constraint which was present in the spot market.

3 Mitigating factors

Certain features of asset markets should make the depressing effect on prices less pronounced.

First, transaction costs in the spot market prevent arbitrageurs from gaining the full benefits of trading and therefore (6) can be violated. In case of housing, broker fees and taxes on real estate gains for example might prevent home owners from selling their houses when the futures price is low. Moreover, the prices of some houses do not have to be perfectly correlated with the housing price index.

Second, not every agent with a positive demand for an asset might actually hold it. In the case of housing, the existence of a minimal amount of housing that one can buy might prevent some agents from demanding housing at all. Such agents could possibly take long positions in the futures market, if minimal purchase values there were substantially lower than those in the spot market. In this case the opening of a futures market would produce competing effects for the direction of the spot market price's change.

Third, assets can generate utility over and above of their investment value. For instance, ownership of a house typically brings more benefits than just the impound rent. Those are sometimes associated with the notion of "pride of ownership", but also with the additional security about the length of time one can stay in a given residence, and with added control over the appearance of the house, etc. One can modify the utility maximization problems (1) and (4) by introducing a term that reflects this additional utility from ownership, for instance by transforming them, respectively, into

$$\begin{aligned}
\max_{m_i, K_i^R \geq 0} \quad & \Big\{ \quad u_{0,i}(W_i - m_i - p_0 K_i^R) \\
& \quad + E_i[u_{1,i}\left(m_i + \tilde{p}_1 K_i^R + r K_i^R \,,\, K_i^R\right)]\Big\}, \\
\max_{m_i,\, K_i^R \geq 0,\, K_i^F} \quad & \Big\{ \quad u_{0,i}(W_i - m_i - p_0 K_i^R) \\
& \quad + E_i[u_{1,i}(m_i + \tilde{p}_1 K_i^R + r K_i^R + (\tilde{p}_1 - F) K_i^F \,,\, K_i^R)] \Big\}.
\end{aligned}$$

We would assume that the marginal utility from owning a house decreases as the amount of house increases.

The so modified problem is substantially different from the simpler problem that we studied in this note. The no-arbitrage condition (6) is no longer valid and long positions in the futures market are no longer perfect substitutes for buying a house. We defer a careful analysis of this more complicated problem, but remark here that the additional utility from owning a house rather than a long futures position could increase as well as decrease the predicted drop in price. The direction in which this effect influences the price change depends on the joint distribution of the preferences for ownership and the beliefs about the spot market price in period 1.

Finally, our model abstracts from time. The predicted price drop might be spread over a period of time beginning before and ending after the opening of the futures market. Before the opening, anticipation of the futures market might already depress an asset's spot price. After the opening, the spot price might not fall immediately as futures markets can be very thin in the beginning.

Appendix A: Constant absolute risk aversion

It is standard in studies of futures markets to assume that agents display CARA utility functions. In this case, we have $u_{0,i}(x) = -e^{-a_{0,i}x}$ and $u_{1,i}(x) = -e^{-a_{1,i}x}$, for positive constants $a_{0,i}$, $a_{1,i}$. Thus,

$$\begin{aligned} & u_{0,i}(W_i - S_i) + E_i[u_{1,i}(S_i + (\tilde{p}_1 - p_0 + r)K_i)] \\ = \; & -e^{-a_{0,i}(W_i - S_i)} + e^{-a_{1,i}S_i} E_i\left[e^{-a_{1,i}(\tilde{p}_1 - p_0 + r)K_i}\right] . \end{aligned}$$

One of the first order conditions for problem (7) is then

$$E_i\left[(\tilde{p}_1 - p_0 + r)e^{-a_{1,i}(\tilde{p}_1 - p_0 + r)K_i}\right] \;=\; 0\,,$$

which can be solved for $K_i(p_0)$.

A standard computation then gives

$$\frac{\partial K_i(p_0)}{\partial p_0} \;=\; -\,\frac{E\left[e^{-a_{1,i}(\tilde{p}_1 - p_0 + r)K_i(p_0)}\right]}{E\left[(\tilde{p}_1 - p_0 + r)^2\; e^{-a_{1,i}(\tilde{p}_1 - p_0 + r)K_i(p_0)}\right]} \;<\; 0 \qquad (11)$$

as required in our arguments.

The solution to problem (2) can now be written as

$$K_i^R(p_0) \;=\; \begin{cases} K_i(p_0) & \text{if } K_i(p_0) > 0\,, \\ 0 & \text{otherwise}\,. \end{cases} \qquad (12)$$

This is so because the first case is equivalent to $E_i[\tilde{p}_1] > p_0 - r$, while the second one is equivalent to $E_i[\tilde{p}_1] \le p_0 - r$. (11) and (12) show that $K_i^R(p_0)$ also satisfy the required conditions in our arguments.

Appendix B: Arbitrage condition

The no-arbitrage condition (6) can be derived from the maximization problem (4) as follows. First, write (4) as

$$\max_{m_i,\, K_i^R \ge 0,\, K_i^F} \left\{ u_{0,i}(W_i - S_i) + E_i[u_{1,i}(S_i + (\tilde{p}_1 - p_0 + r)K_i^R + (\tilde{p}_1 - F)K_i^F)] \right\}.$$

In case $F < p_0 - r$, then decreasing K_i^R and correspondingly increasing K_i^F always increases utility. Therefore all agents will optimize with $K_i^R = 0$. This is incompatible with the market clearing condition $\sum_i K_i^R = H$.

In case $F > p_0 - r$, then increasing K_i^R and correspondingly decreasing K_i^F always increases utility. Therefore there will be no equilibrium value for K_i^R and K_i^F.

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