MATH 3C (Spring 2008)
Instructor: Roberto Schonmann
Midterm Exam

Last Name:

First and Middle Names:

Signature:

UCLA id number (if you are an extension student, say so):

Discussion section in which you are enrolled (Day, time, name of TA, 1A ... 2F):

Using a pen, provide the information asked above, and also write your name on the top of each page of this exam. When the instructions to a question ask you to explain your answer, you should show your work and explain what you are doing carefully; this is then more important than just finding the right answer. Please, write clearly and make clear what your solution and answer to each problem is. When you continue on another page indicate this clearly. To cancel anything from your solution, erase it or cross it out. You are not allowed to sit close to students with whom you have studied for this exam, or to your friends.

Good Luck !

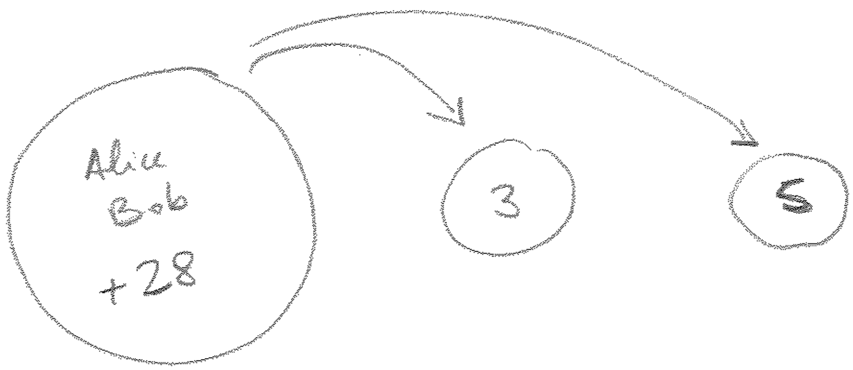
Question	1	2	3	4	5	6	7	8	9	10	Total
Score											

1) (10 points) In a group of 11 students, 7 are freshmen and 4 are sophomores. In how many ways can the students in this group form a waiting line, if the first two in the line and the last two in the line should be sophomores? (No need to compute factorials, powers, permutations and combinations.) (No need to show and explain work.)

$$\underbrace{\underline{4}\ \ \underline{3}}_{\text{soph.}}\ \ \underbrace{\underline{7}\ \ \underline{6}\ \ \underline{5}\ \ \underline{4}\ \ \underline{3}\ \ \underline{2}\ \ \underline{1}}_{\text{fresh.}}\ \ \underbrace{\underline{2}\ \ \underline{1}}_{\text{soph.}}$$

Answer : $4! \times 7!$

2) (10 points) A group of 30 people includes Alice and Bob. A team of 3 people and a team of 5 people are selected from this group of 30 people (no one can be selected for both teams). In how many ways can the selection be made if Alice and Bob should each be selected for one of these two teams (not for the the same team). (No need to compute factorials, powers, permutations and combinations.) (No need to show and explain work.)



not Alice nor Bob

in team of 3

Alice in team of 3, Bob in team of 5 : $\binom{28}{2}\binom{28-2}{4}$

Team of 3 Team of 5

not Alice or Bob

Same for Bob in team of 3 and Alice in team of 4

<u>Answer:</u> $2 \times \binom{28}{2}\binom{26}{4}$

3) (10 points) Assume that $P(A) = 0.3$, $P(B) = 0.4$, $P(A \cup B) = 0.6$. Find $P(A^c \cup B^c)$. (Give answer in decimal form.) (Show your work.)

$$P(A^c \cup B^c) = P((A \cap B)^c)$$

$$= 1 - P(A \cap B)$$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= 0.3 + 0.4 - 0.6 = 0.1$$

So $P(A^c \cup B^c) = 1 - 0.1 = \boxed{0.9}$

4) (10 points) Five cards are drawn with no replacement from a standard deck of 52 cards. What is the probability of having a full house, i.e., 3 cards of one value and two cards of another value (for instance 888JJ). (No need to compute factorials, powers, permutations and combinations.) (No need to show and explain work.)

value of 3 cards

value of 2 cards

suits of 3 cards

suits of 2 cards

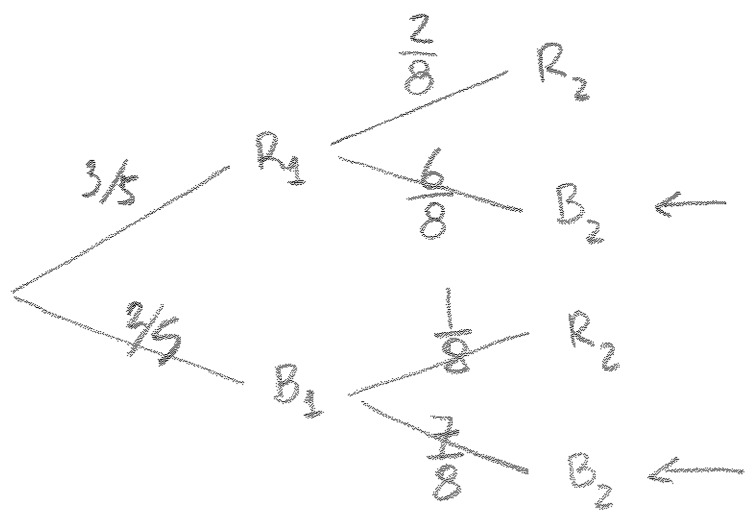
$$\frac{13 \times 12 \times \binom{4}{3} \times \binom{4}{2}}{\binom{52}{5}}$$

5) (10 points) A regular deck of cards is missing the ace of spades, so that it has only 51 cards. If 4 cards are taken at random without replacement from this deck, what is the probability that they are of different suits? (No need to compute factorials, powers, permutations and combinations.) (No need to show and explain work.)

$$\frac{\underset{\text{hearts}}{13} \times \underset{\text{clubs}}{13} \times \underset{\text{diamonds}}{13} \times \underset{\text{spades}}{12}}{\binom{51}{4}} = \frac{13^3 \times 12}{\binom{51}{4}}$$

Must have exactly one card of each suit.

6) (10 points) We have 2 boxes. Box I contains 3 red balls and 2 blue balls. Box II contains 1 red ball and 6 blue balls. One ball is taken at random from box I and placed in box II. Afterwards one ball is taken at random from box II. What is the probability that the ball selected from box II is blue? (Give answer as a fraction or in decimal form.) (Show your work.)



$R_1 =$ ball from box I is red

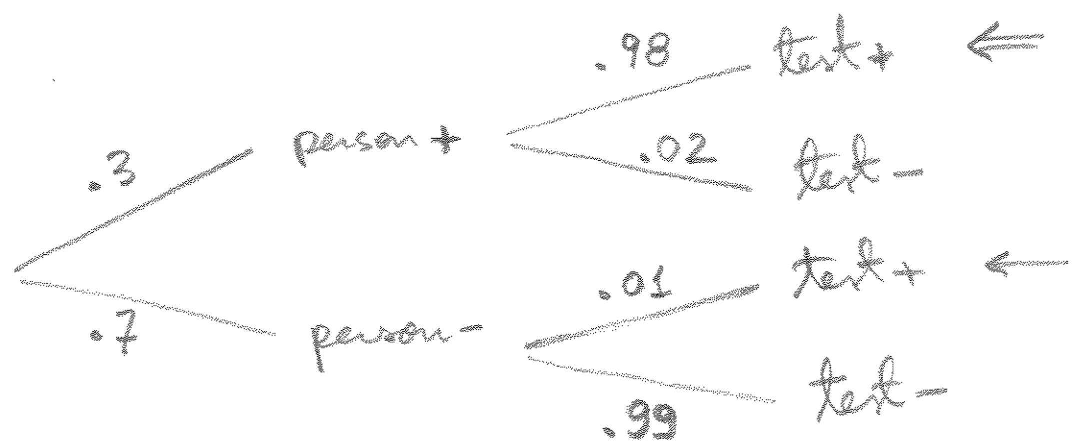
$B_1 =$ " " " " " blue

$R_2 =$ " " " II " red

$B_2 =$ " " " " " blue

$$P(B_2) = \frac{3}{5}\times\frac{6}{8} + \frac{2}{5}\times\frac{7}{8} = \frac{9+7}{20} = \frac{16}{20} = \boxed{\frac{4}{5} = .8}$$

7) (10 points) A screening test for a disease shows a false positive with probability 1% and a false negative with probability 2%. In the population 30% of people have that disease. Given that someone tested positive for the disease, what is the probability that he/she has the disease? (Provide a numerical answer in decimal form, or as a percentage.) (Show your work.)



$$P(\text{person}+ \mid \text{test}+) = \frac{P(\text{person}+, \text{test}+)}{P(\text{test}+)}$$

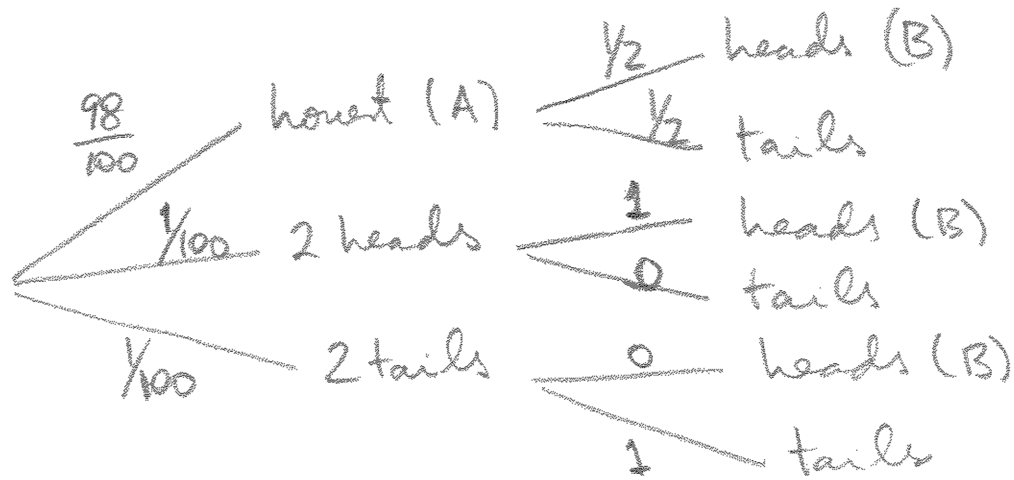
$$= \frac{.3 \times .98}{.3 \times .98 + .7 \times .01} = \frac{.294}{.294 + .007} \cong .98 = 98\%$$

8) (10 points) A box contains 98 honest coins, 1 coin with two heads and one coin with 2 tails. One coin is picked at random from this box and flipped. Consider the events:

A: the coin picked at random is honest.

B: the coin that was flipped showed heads.

Are the events A and B independent? (Explain your answer.)



tree diagram

$$P(AB) = \frac{98}{100} \times \frac{1}{2} = \frac{49}{100}$$

tree diagram

$$P(A) = \frac{98}{100} \qquad P(B) = \frac{98}{100} \times \frac{1}{2} + \frac{1}{100} \times 1 = \frac{1}{2}$$

So $P(AB) = P(A) \cdot P(B)$

$\Rightarrow$ Yes, they are independent.

9) (10 points) A coin is flipped three times. Consider the random variable V that gives the numbers of heads shown minus the number of tails shown. Find the probability mass function of V. (Give answer as a fraction or in decimal form.) (Show your work.)

Ω	V
hhh	$3-0=3$
hht	$2-1=1$
hth	$2-1=1$
htt	$1-2=-1$
thh	$2-1=1$
tht	$1-2=-1$
tth	$1-2=-1$
ttt	$0-3=-3$

$$P_V(-3) = \frac{1}{8} = .125$$

$$P_V(-1) = \frac{3}{8} = .375$$

$$P_V(1) = \frac{3}{8} = .375$$

$$P_V(3) = \frac{1}{8} = .125$$

10) (10 points) A coin is flipped 4 times. Consider the random variable W defined as the number of times that the coin shows heads immediately after having showed tail. (For instance $W(hhth) = 1$, $W(thht) = 1$, $W(thth) = 2$, $W(hhhh) = 0$, $W(tthh) = 1$.) Compute the mean and the standard deviation of W. (Provide numerical answers in decimal form.) (Show your work.)

Ω	W
hhhh	0
hhht	0
hhth	1
hhtt	0
hthh	1
htht	1
htth	1
httt	0
thhh	1
thht	1
thth	2
thtt	1
tthh	1
ttht	1
ttth	1
tttt	0

$$P_W(0) = \frac{5}{16}$$

$$P_W(1) = \frac{10}{16}$$

$$P_W(2) = \frac{1}{16}$$

$$EW = 1 \times \frac{10}{16} + 2 \times \frac{1}{16} = \frac{12}{16} = \boxed{.75}$$

$$E(W^2) = 1^2 \times \frac{10}{16} + 2^2 \times \frac{1}{16} = \frac{14}{16} = .875$$

$$Var(W) = EW^2 - (EW)^2$$

$$= .875 - (.75)^2$$

$$= .3125$$

$$\sigma_W = \sqrt{Var(W)} \approx \boxed{.559}$$