MATH 3C (Spring 2007, Lecture 2)
Instructor: Roberto Schonmann
Midterm Exam

Last Name:

First and Middle Names: Solutions

Signature:

UCLA id number (if you are an extension student, say so):

Circle the discussion section in which you are enrolled:

2A (T 10, Akemi Kashiwada)

2B (R 10, Akemi Kashiwada)

2C (T 10, Wenhua Gao)

2D (R 10, Wenhua Gao)

2E (T 10, Ilhwan Jo)

2F (R 10, Ilhwan Jo)

Using a pen, provide the information asked above, and also write your name on the top of each page of this exam. When the instructions to a question ask you to explain your answer, you should show your work and explain what you are doing carefully; this is then more important than just finding the right answer. You can use the blank pages at the end of the exam as scratch paper or if you need space to finish the solution to a question. Please, make clear what your solution and answer to each problem is. When you continue on another page indicate this clearly. You are not allowed to sit close to students with whom you have studied for this exam, or to your friends.

Good Luck !

Question	1	2	3	4	5	6	Total
Score							

1) (10 points) A group of people has 4 adults and 8 children. In how many ways can we select 6 people from this group, with exactly one or two adults being selected? (No need to compute factorials, powers, permutations and combinations.) (No explanation needed, just the answer is enough.)

$\binom{4}{1}\binom{8}{5} + \binom{4}{2}\binom{8}{4}$

Exactly 1 adult

Exactly 2 adults

2) (10 points) Roll a die 10 times. What is the probability that at least one face shown is a 4? (No need to compute factorials, powers, permutations and combinations.) (No explanation needed, just the answer is enough.)

$P(\text{at least one "4"})$

$= 1 - P(\text{no "4"}) = 1 - \left(\frac{5}{6}\right)^{10}$

3) (10 points) In a group of students, 7 are freshmen, and 9 are sophomores. If they form a waiting line in a random order, what is the probability that all the freshmen will be ahead of all the sophomores in the line? (No need to compute factorials, powers, permutations and combinations.) (No explanation needed, just the answer is enough.)

$$\frac{7! \times 9!}{16!}$$

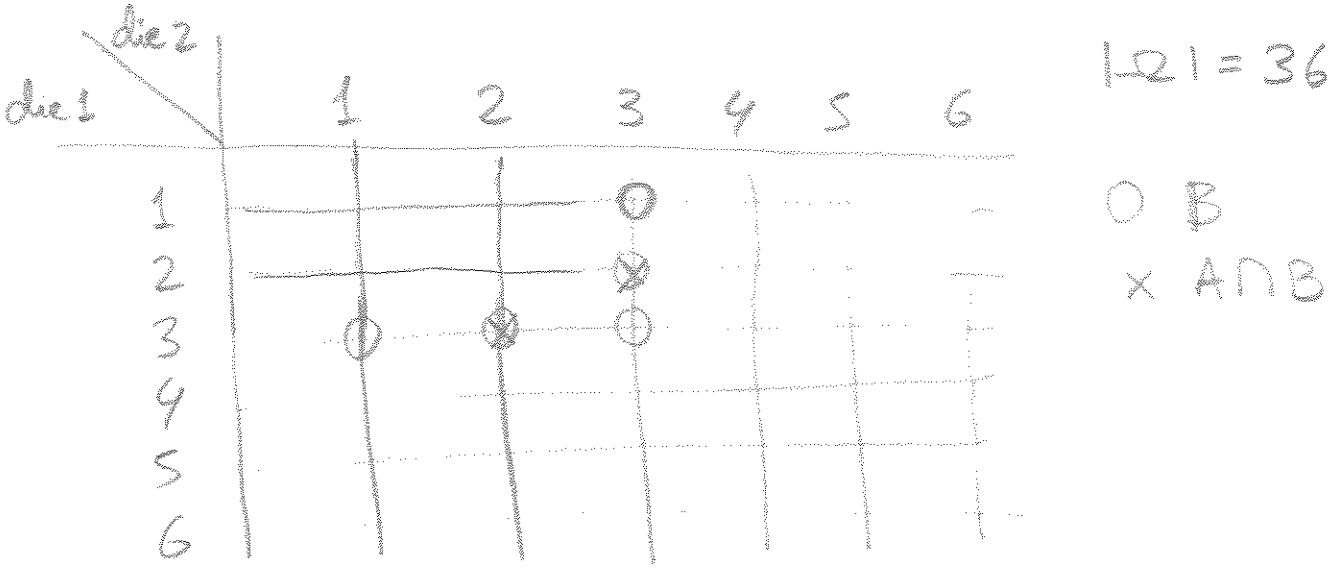
Ω : order $7+9=16$ people $\Rightarrow |\Omega| = 16!$

favorable cases : order freshmen $\Rightarrow 7!$
order sophomores $\Rightarrow 9!$

4) (10 points) You roll two fair dice. Compute the conditional probability that the sum of the faces shown is 5, given that the maximum of the faces shown is 3 (Give answer as a fraction or in decimal form.) (Explain your answer carefully.)

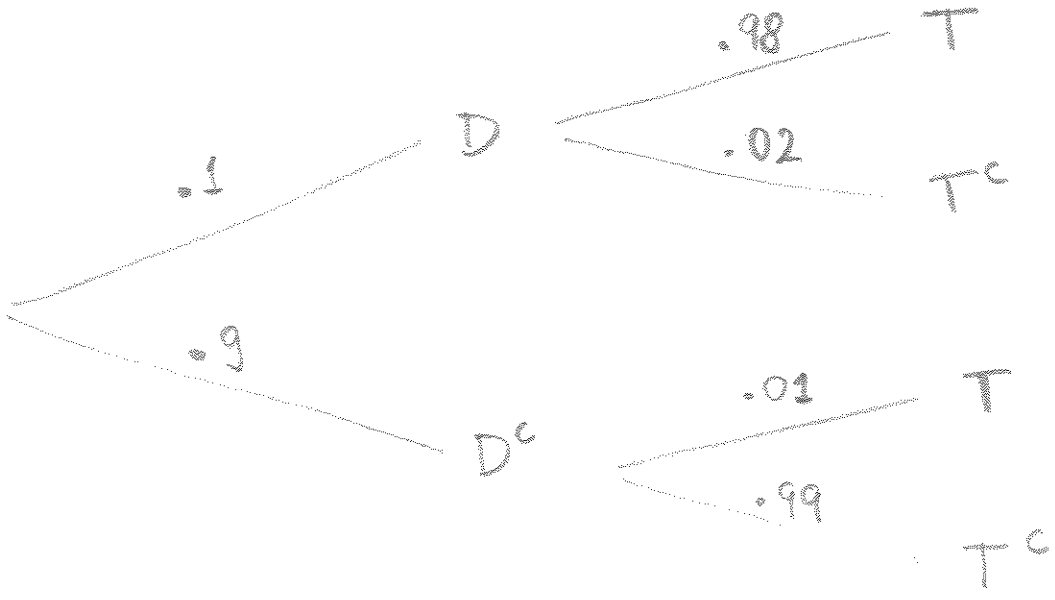
A: sum is 5.

B: maximum of faces is 3.



$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{2/36}{5/36} = \frac{2}{5}$$

5) (10 points) A screening test for a disease shows a false positive with probability 1% and a false negative with probability 2%. In the population 10% of people have that disease. Given that someone tested negative for the disease, what is the probability that he/she has the disease? (Provide a numerical answer in decimal form, or as a percentage.) (Explain your answer carefully.)

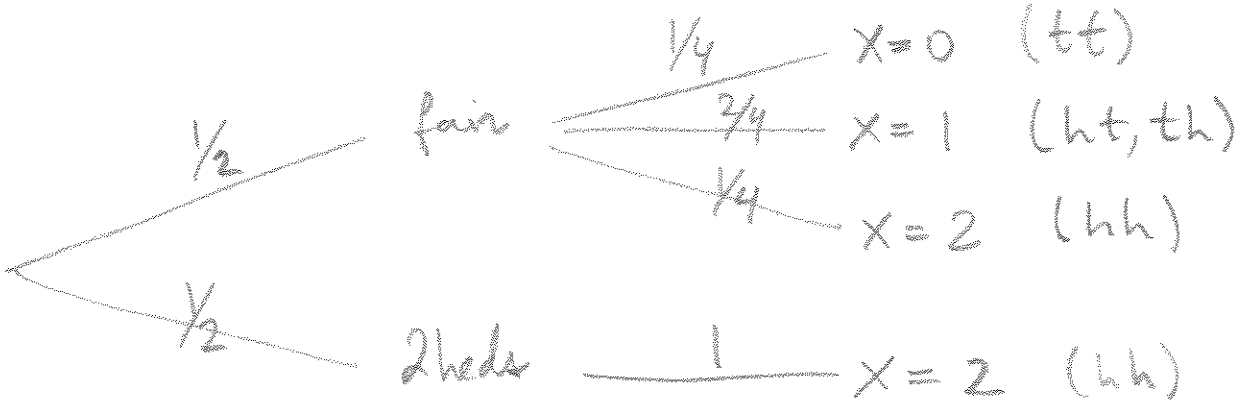


$$P(D \mid T^c) = \frac{P(D T^c)}{P(T^c)} = \frac{.1 \times .02}{.1 \times .02 + .9 \times .99}$$

$$= \frac{2}{2 + 9 \times 99} \approx 0.00224$$

$$= .224\%$$

6) (10 points) A box contains a fair coin and a coin with two heads. One coin is selected at random from this box and flipped two times. Let X be the random variable that gives the number of heads in the two flips of the coin. Find the probability mass function of X. (the values that you compute should be expressed as fractions, or in decimal form.) (Explain your answer carefully.)



$$p_X(0) = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

$$p_X(1) = \frac{1}{2} \times \frac{2}{4} = \frac{1}{4}$$

$$p_X(2) = \frac{1}{2} \times 1 + \frac{1}{2} \times \frac{1}{4} = \frac{5}{8}$$