MATH 3C (Spring 2007, Lecture 1)
Instructor: Roberto Schonmann
Midterm Exam

Last Name:

First and Middle Names: Solutions

Signature:

UCLA id number (if you are an extension student, say so):

Circle the discussion section in which you are enrolled:

1A (T 9, Matthew Keegan)   1B (R 9, Matthew Keegan)

1C (T 9, Yan Wang)   1D (R 9, Yan Wang)

1E (T 9, Christopher McKinlay)   1F (R 9, Christopher McKinlay)

Using a pen, provide the information asked above, and also write your name on the top of each page of this exam. When the instructions to a question ask you to explain your answer, you should show your work and explain what you are doing carefully; this is then more important than just finding the right answer. You can use the blank pages at the end of the exam as scratch paper or if you need space to finish the solution to a question. Please, make clear what your solution and answer to each problem is. When you continue on another page indicate this clearly. You are not allowed to sit close to students with whom you have studied for this exam, or to your friends.

Good Luck !

Question	1	2	3	4	5	6	Total
Score							

1) (10 points) In a group of students, 8 are freshmen, 5 are sophomores, 3 are juniors and 6 are seniors. In how many ways can the students in this group form a waiting line, if all seniors should be ahead of all juniors, all juniors should be ahead of all sophomores, and all sophomore should be ahead of all freshmen? (No need to compute factorials, powers, permutations and combinations.) (No explanation needed, just the answer is enough.)

6!   3!   5!   8!

Seniors	Juniors	Sophomores	Freshmen

Answer = 6! × 3! × 5! × 8!

2) (10 points) Roll a die 2 times. What is the conditional probability that the sum of the faces shown is 6, given that both faces shown are even numbers? (Give answer as a fraction or in decimal form.) (Explain your answer carefully.)

A: sum of faces shown is 6.

B: both faces shown are even.

$|\Omega| = 36$

O A

x A ∩ B

From diagram:

$|A| = 9$

$|A \cap B| = 2$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{2/36}{9/36} = \frac{2}{9}$$

3) (10 points) A group of people has 5 adults and 9 children. From this group, 6 people are selected at random. Compute the probability that at least one adult is selected. (No need to compute factorials, powers, permutations and combinations.) (No explanation needed, just the answer is enough.)

$$1 - P(\text{No adult selected})$$

$$= 1 - \frac{\binom{9}{6}}{\binom{14}{6}}$$

4) (10 points) You are dealt two cards from a standard deck of 52 cards. Consider the events A that the two cards are spades, and B that the two cards are aces. Are the events A and B independent? (Explain your answer carefully.)

$A \cap B = \{ 2 \text{ aces of spades} \}$
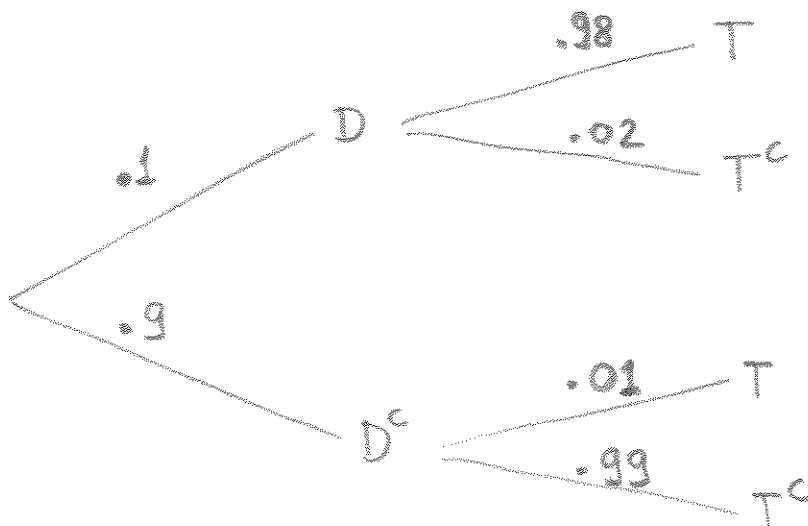
$= \emptyset$ (Because there is only one ace of spades)

So $P(A \cap B) = 0$

$P(A) = \dfrac{\binom{13}{2}}{\binom{52}{2}} \quad , \quad P(B) = \dfrac{\binom{4}{2}}{\binom{52}{2}}$

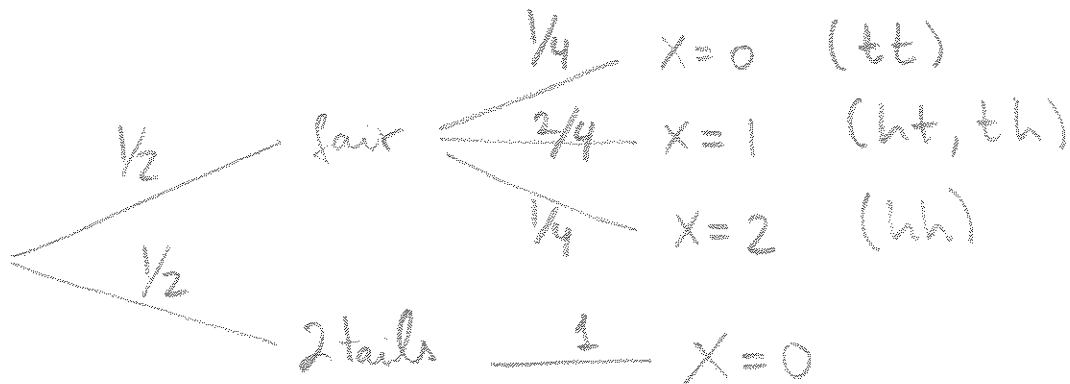Therefore $P(A \cap B) \neq P(A) P(B)$

Answer : No.

5) (10 points) A screening test for a disease shows a false positive with probability 1% and a false negative with probability 2%. In the population 10% of people have that disease. Given that someone tested positive for the disease, what is the probability that he/she has the disease? (Provide a numerical answer in decimal form, or as a percentage.) (Explain your answer carefully.)

$$.1 \quad D \quad \overset{.98}{\longrightarrow} T$$
$$D \quad \overset{.02}{\longrightarrow} T^c$$
$$.9 \quad D^c \quad \overset{.01}{\longrightarrow} T$$
$$D^c \quad \overset{.99}{\longrightarrow} T^c$$

D = person has disease T = person tests positive

$$P(D \mid T) = \frac{P(DT)}{P(T)} = \frac{.1 \times .98}{.1 \times .98 + .9 \times .01}$$

$$= \frac{98}{98 + 9} = \frac{98}{107} \approx \boxed{.916}$$

6) (10 points) A box contains a fair coin and a coin with two tails. One coin is selected at random from this box and flipped two times. Let X be the random variable that gives the number of heads in the two flips of the coin. Find the probability mass function of X. (the values that you compute should be expressed as fractions, or in decimal form.) (Explain your answer carefully.)

$$\frac{1}{4} \quad X=0 \quad (tt)$$

$$\frac{1}{2} \quad \text{fair} \quad \frac{2}{4} \quad X=1 \quad (ht, th)$$

$$\frac{1}{4} \quad X=2 \quad (hh)$$

$$\frac{1}{2}$$

$$\text{2 tails} \quad \underline{1} \quad X=0$$

$$P_X(0) = \frac{1}{2} \times \frac{1}{4} + \frac{1}{2} = \frac{5}{8}$$

$$P_X(1) = \frac{1}{2} \times \frac{2}{4} = \frac{1}{4}$$

$$P_X(2) = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$