MATH 167 (Fall 2006)
Instructor: Roberto Schonmann
Midterm Exam

Last Name:

First and Middle Names: Solutions

Signature:

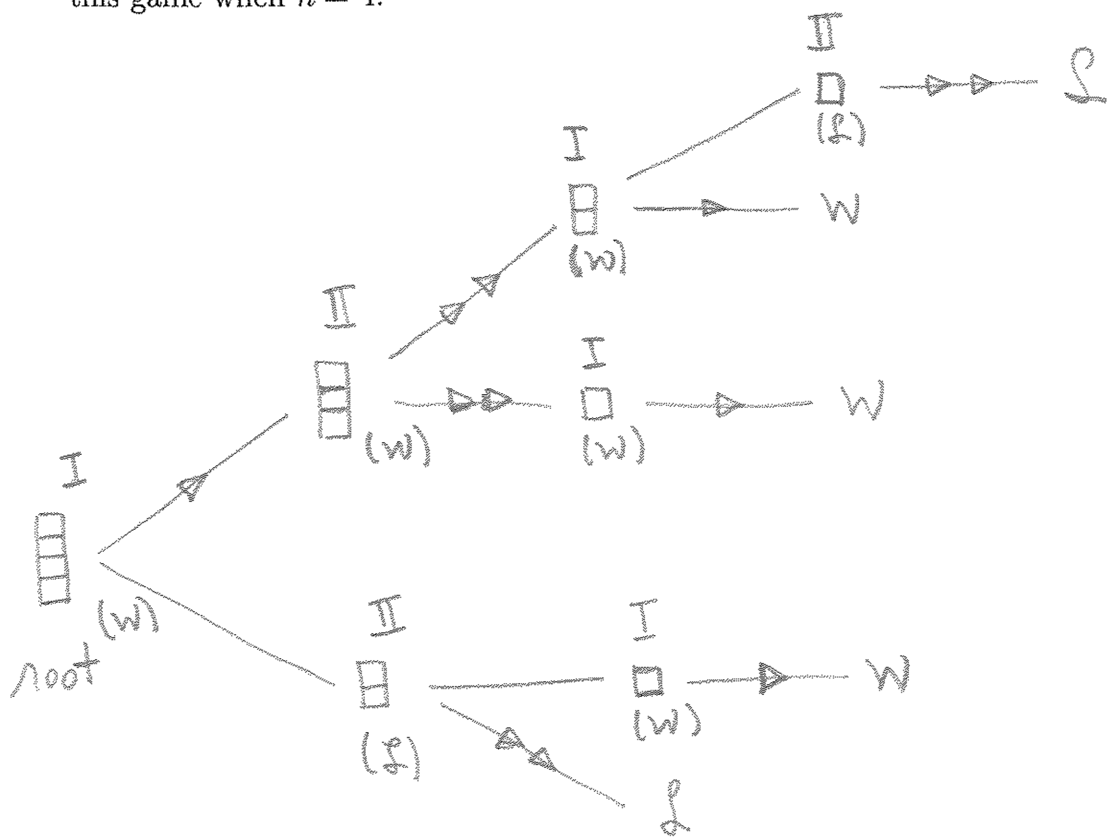
UCLA id number (if you are an extension student, say so):

Provide the information asked above and write your name on the top of each page using a pen. You should show your work and explain what you are doing; this is more important than just finding the right answer. You can use the blank pages as scratch paper or if you need space to finish the solution to a question. Please, make clear what your solution and answer to each problem is. When you continue on another page indicate this clearly. You are not allowed to sit next to students with whom you have been studying for this exam or to your friends.

Good Luck !

Question	1	2	3	4	5	Total
Score						

1) (10 points) In a version of the game "pick-up-bricks" two players alternate taking 1 or 2 bricks from a pile that starts with n bricks. The players are called I and II, with I being the one who moves first. The winner is the player who removes the last brick. Draw the game tree and compute the value of this game when $n = 4$.



value = W (by backward induction)

2) (10 points) Give an example of a 2×2 matrix with no saddle point.

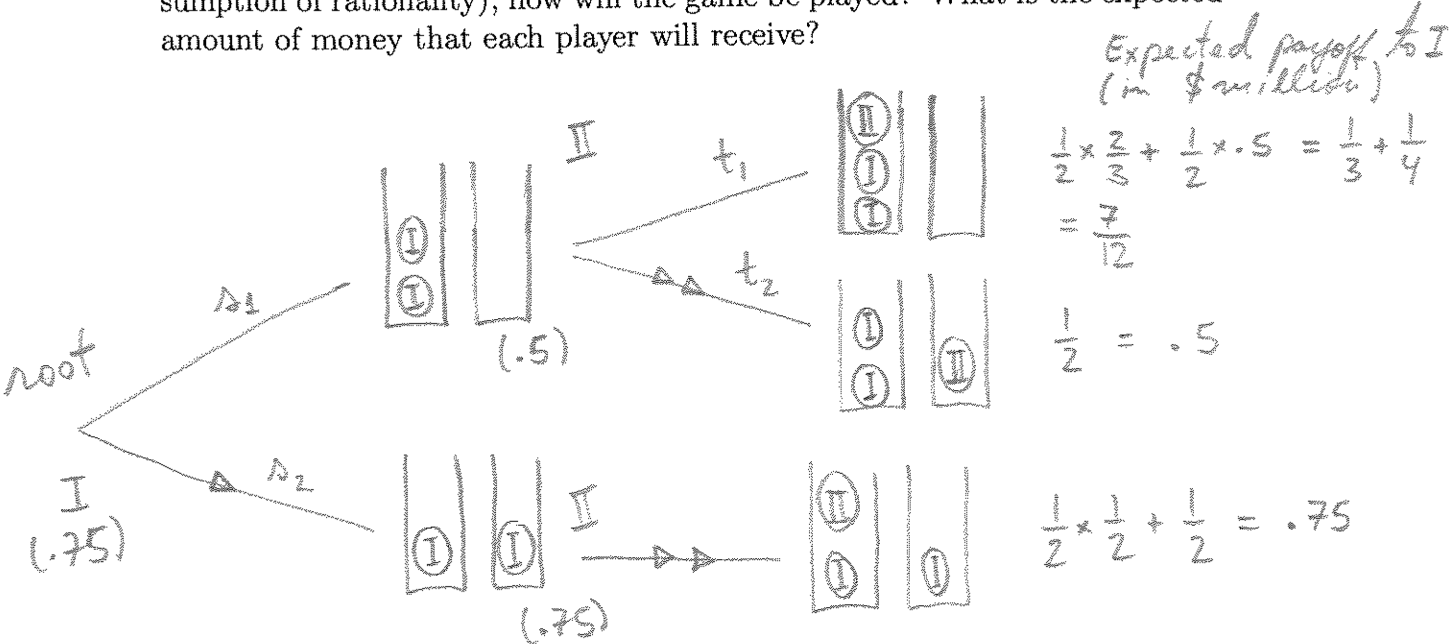
$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

3) (10 points) Define Nash equilibria for games with two players.

A pair of strategies (s,t) is a Nash equilibrium, if s is a best response to t and t is a best response to s.

4) (10 points) In this game player I has two chips marked with I and she moves first, placing them in any way she wants in two identical boxes. Player II has only one chip marked with II, and she moves after player I, placing her chip in either one of the two boxes (which are open with their content visible). A referee then selects a box at random and if this box is not empty she picks a chip at random from this box. The referee then pays the players according to the following rules. 1) If the box she picked at random was empty, each player receives $ 500,000. 2) If the box she picked at random was not empty, so that she picked a chip from it, player i receives $ 1,000,000 and the other player receives nothing, where i is the mark on the picked chip.

For the case in which the two players are risk neutral (and with the assumption of rationality), how will the game be played? What is the expected amount of money that each player will receive?



Risk-neutral ⇒ Expected utility = expected monetary payoff

Expected payoff to player II = 1 − expected payoff to player I

(Game is strictly competitive).

Answer: Using backward induction (see picture), Player I places one chip in each box. Player II places a chip in either box. I expects $750,000, II expects $250000

5) (10 points) Consider the game from the previous problem, but assume that the players are no longer supposed to be risk neutral. To assess the level of risk aversion of the players they are asked which lottery with only possible prizes being \$ 1,000,000 and \$0 each one of them finds as attractive as receiving \$ 500,000 for sure. Player I responds that it would be the one in which the probability of winning the \$ 1,000,000 is 0.8. Player II responds that it would be the one in which the probability of winning the \$ 1,000,000 is 0.7. Both players know this fact about the other. Represent the game in this case in bimatrix (strategic) form and find the way in which the players will play it, using elimination of dominated strategies (and the assumption of rationality).

Strategies: I: s_1, s_2 II: t_1, t_2 (see picture in previous page)

Risk-aversion: $q_I = .8$, $q_{II} = .7$

Expected utilities: $(s_1, t_1) \to$ I: $\frac{1}{2}\times\frac{2}{3} + \frac{1}{2}\times(.8) = \frac{1}{3} + .4 \cong .733$

II: $\frac{1}{2}\times\frac{1}{3} + \frac{1}{2}\times(.7) = \frac{1}{6} + .35 \cong .513$

$(s_1, t_2) \to$ I: $.5$, II: $.5$

$(s_2, t_1 \text{ or } t_2) \to$ I: $.75$, II: $.25$

	t_1	t_2
s_1	.733, .513	.5, .5
s_2	.75, .25	.75, .25

eliminate t_2

then eliminate s_1

Answer: play s_2, t_1