MATH 167 (Fall 2006)
Instructor: Roberto Schonmann
Final Exam

Last Name:

First and Middle Names: Solutions

Signature:

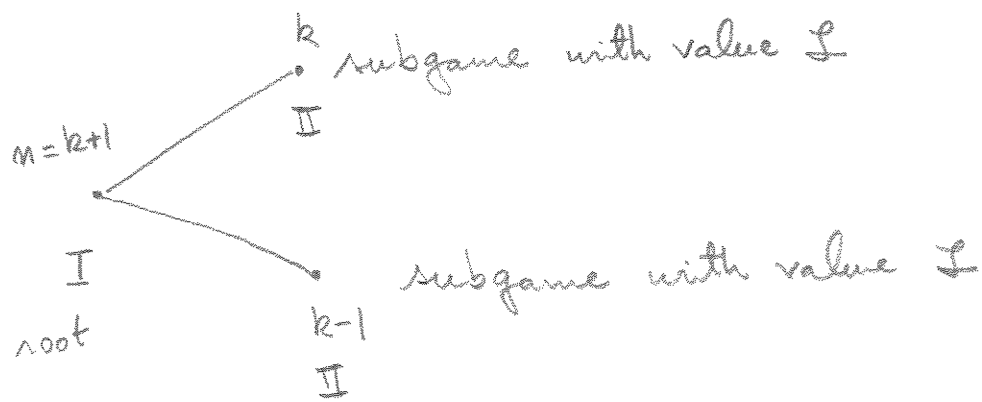
UCLA id number (if you are an extension student, say so):

Provide the information asked above and write your name on the top of each page using a pen. You should show your work and explain what you are doing; this is more important than just finding the right answer. You can use the blank pages as scratch paper or if you need space to finish the solution to a question. Please, make clear what your solution and answer to each problem is. When you continue on another page indicate this clearly. You are not allowed to sit next to students with whom you have been studying for this exam or to your friends.

Good Luck !

Question	1	2	3	4	5	6	7	8	Total
Score									

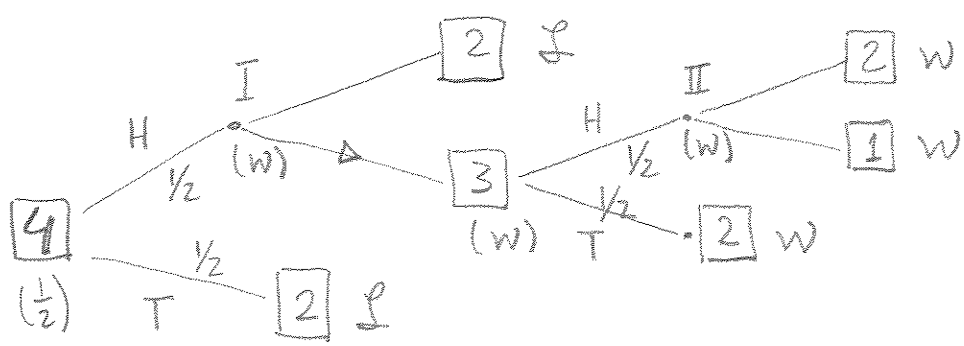
1) (10 points) In a version of the game "pick-up-bricks" two players alternate taking 1 or 2 bricks from a pile that starts with n bricks. The players are called I and II, with I being the one who moves first. The winner is the player who removes the last brick. You are told what the value of this game is when $n = 1, 2, 3, ..., k-1, k$, for some k (for each n the value is $\mathcal{W}$, or $\mathcal{L}$, meaning that player I, or Player II, respectively, can assure a win in the game). If the value is $\mathcal{W}$ when $n = k-1$ and when $n = k$, what can you conclude the value to be when $n = k+1$?



Player I can remove 1 or 2 bricks, leaving k or $k-1$ bricks. This means that when player II moves now, she has a winning strategy (because we are given that with $n = k$ or $k-1$ the value of the game is $\mathcal{W}$). Therefore with $n = k+1$ the value is $\underline{\underline{v = \mathcal{L}}}$.

2) (10 points) The game "pick-up-bricks" from the previous problem is now modified in the following way. When it is a player's turn, a fair coin is tossed; if it shows heads, the player can remove 1 or 2 bricks; but if the coin shows tails, then the player has to remove 2 bricks (unless there was only one brick left, in which case he can remove that single brick). As before, the winner is the player who removes the last brick. Suppose that the game starts with $n = 4$ bricks. The payoff to each player is the probability that this player wins. What is the probability that player I wins the game when both players use the strategies that result from backwards induction?

It is clear that when this game starts with 1 or 2 bricks, the first player wins by removing all bricks, regardless of the toss of the coin. This simplifies the tree below.



$$v = \frac{1}{2} \times 1 + \frac{1}{2} \times 0 = \frac{1}{2}$$

3) (10 points) If possible give an example of a 2×2 matrix with four saddle points and an example of a 2×2 matrix with exactly 3 saddle points. If either of these tasks is impossible, explain clearly why.

4 saddle pts : $\begin{array}{|c|c|} \hline 0 & 0 \\ \hline 0 & 0 \\ \hline \end{array}$

3 saddle pts : impossible.

(s,t) is a saddle pt if and only if $\underline{m} = \overline{m}$ and s and t are security strategies in pure strategies (MM strategies).

So the number of saddle pts, when $\underline{m} = \overline{m}$ is $A \times B$, where $A = \#$ of MM strategies for player I, $B = \#$ of MM strategies for player II. A and B can be 1, 2, so $A \times B = 1$ or 2 or 4.

4) (10 points) In this game player I has two chips marked with I and she moves first, placing them in any way she wants in two identical boxes. Player II has only one chip marked with II, and she moves after player I, placing her chip in either one of the two boxes (which are open with their content visible). A referee then selects a box at random and if this box is not empty she picks a chip at random from this box. The referee then pays the players according to the following rules. 1) If the box she picked at random was empty, each player receives \$500,000. 2) If the box she picked at random was not empty, so that she picked a chip from it, player i receives \$1,000,000 and the other player receives nothing, where i is the mark on the picked chip.

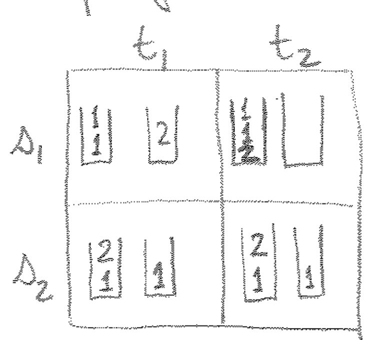
Before the game starts, the level of risk aversion of each player is assessed in the following way. They are asked which lottery with only possible prizes being \$1,000,000 and \$0 each one of them finds as attractive as receiving \$500,000 for sure. Player I responds that it would be the one in which the probability of winning the \$1,000,000 is 0.5. Player II responds that it would be the one in which the probability of winning the \$ 1,000,000 is 0.9. Both players know this fact about the other.

a) Represent the game as a 2×2 bimatrix game, using the following notation.
Player I has strategies
s_1: both chips in same box,
s_2: chips in different boxes.
Player II has strategies
t_1: if player I left a box empty, place her chip in that box,
t_2: if player I left a box empty, place her chip in the same box that has Player's I two chips.

b) How will player I play this game?

Let's use the utility scale with $u(\$1000000) = 1$, $u(\$0) = 0$ for both players.

a)



$\xrightarrow[\text{(see below)}]{\text{utilities}}$

	t_1	t_2
s_1	$\frac{1}{2}, \frac{1}{2}$	$\frac{7}{12}, \frac{37}{60}$
s_2	$\frac{2}{3}, \frac{1}{3}$	$\frac{2}{3}, \frac{1}{3}$

$(s_1, t_1) \to \begin{cases} \text{I}: \frac{1}{2}\times 1 + \frac{1}{2}\times 0 = \frac{1}{2} \\ \text{II}: \frac{1}{2}\times 0 + \frac{1}{2}\times 1 = \frac{1}{2} \end{cases}$

$(s_1, t_2) \to \begin{cases} \text{I}: \frac{1}{2}\times\frac{2}{3}\times 1 + \frac{1}{2}\times .5 = \frac{1}{3} + \frac{1}{4} = \frac{7}{12} \\ \text{II}: \frac{1}{2}\times\frac{1}{3}\times 1 + \frac{1}{2}\times .9 = \frac{1}{6} + \frac{9}{20} = \frac{37}{60} \end{cases}$

(s_2, t_1) or $(s_2, t_2) \to \begin{cases} \text{I}: \frac{1}{2}\times\frac{1}{3}\times 1 + \frac{1}{2}\times 1 = \frac{1}{6} + \frac{1}{2} = \frac{4}{6} = \frac{2}{3} \\ \text{II}: \frac{1}{2}\times\frac{2}{3}\times 1 + \frac{1}{2}\times 0 = \frac{1}{2} \end{cases}$

b)

	t_1	t_2
s_1	$\frac{1}{2}, \frac{1}{2}$	$\frac{7}{12}, \frac{37}{60}$
s_2	$\frac{2}{3}, \frac{1}{3}$	$\frac{2}{3}, \frac{1}{3}$

$$\frac{1}{2} < \frac{2}{3} \quad , \quad \frac{7}{12} < \frac{8}{12} = \frac{2}{3}$$

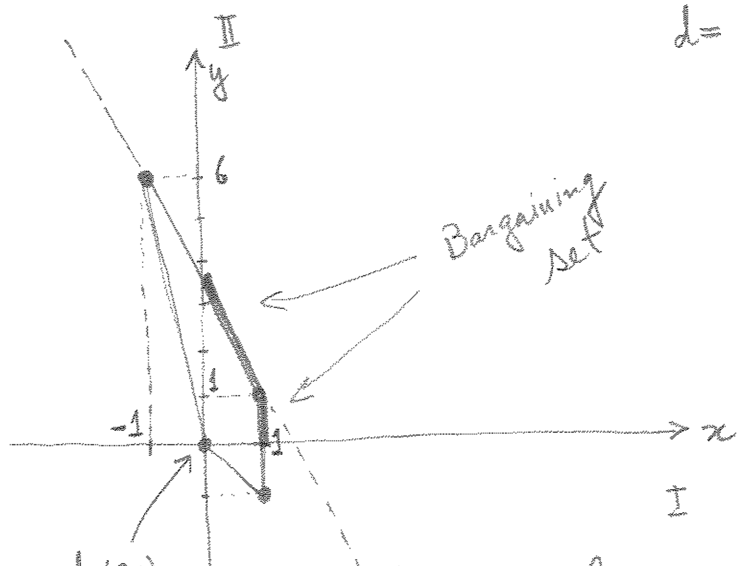
so s_1 is strongly dominated by s_2

and player I will play $\underline{\underline{s_2}}$.

5) (10 points) Consider the following bimatrix game.

$$\begin{bmatrix} (\ 0, 0) & (1, -1) \\ (-1, 6) & (1, \ \ 1) \end{bmatrix}$$

Compute the payoff to each player if they use Nash's standard bargaining solution (standard means that the bargaining powers are equal: $\alpha = \beta = 1/2$), with the understanding that the disagreement point is the only Nash equilibrium in pure strategies for this game.



d= disagreement pt = NE pure str.

$$\begin{bmatrix} \boxed{0}, \boxed{0} & \boxed{1}, -1 \\ -1, \boxed{6} & \boxed{1}, 1 \end{bmatrix}$$

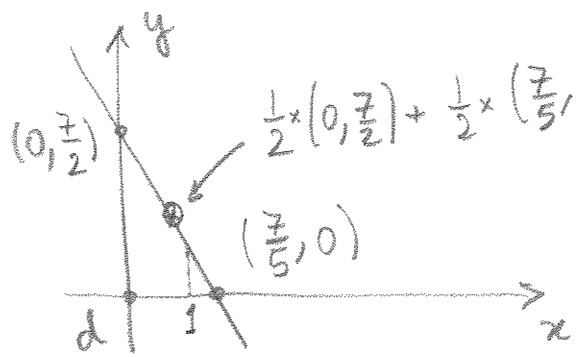
So $d = (0,0)$

L $\longrightarrow$ equation $\quad \dfrac{x-1}{y-1} = \dfrac{-1-1}{6-1}$

$\therefore \; 5x - 5 = -2y + 2$

$\therefore \; 5x + 2y - 7 = 0$

Try a solution ⊗ on L:



$\frac{1}{2} \times (0, \frac{7}{2}) + \frac{1}{2} \times (\frac{7}{5}, 0) = (\frac{7}{10}, \frac{7}{4})$

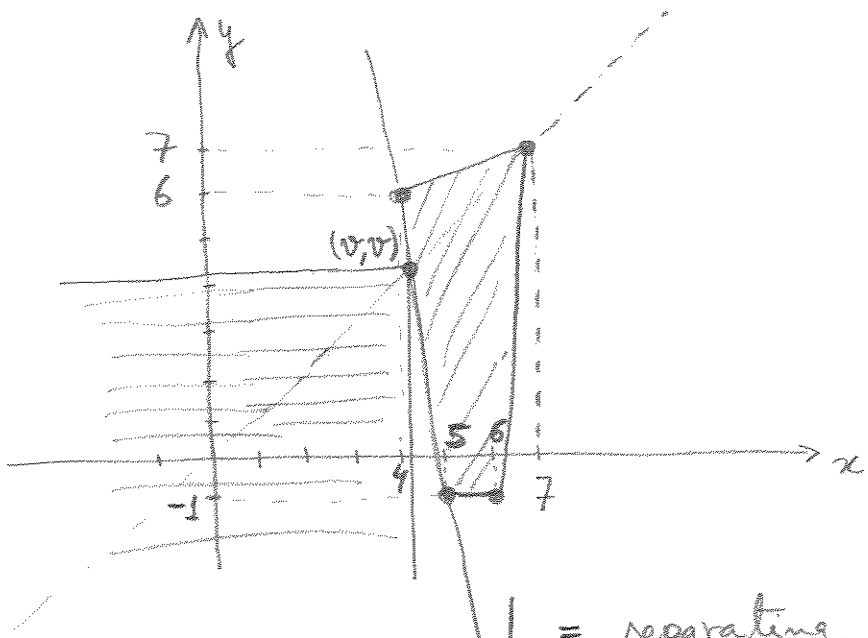
Does it belong to bargaining set?

Yes, since $0 < \frac{7}{10} < 1$.

$\boxed{\text{Answer: } \frac{7}{10} \text{ to I}, \ \frac{7}{4} \text{ to II}}$

6) (10 points) Use the method of the separating line to compute the value of the matrix game below. Do not eliminate dominated strategies.

$$\begin{bmatrix} 7 & 5 & 6 & 4 \\ 7 & -1 & -1 & 6 \end{bmatrix}$$



L = separating line (through (5,-1), (4,6))

$$\frac{x-5}{y-(-1)} = \frac{4-5}{6-(-1)} \quad \therefore \quad 7x - 35 = -y - 1$$

$$\therefore \quad 7x + y = 34$$

$$(v,v) \in L \quad \Rightarrow \quad 7v + v = 34 \quad \Rightarrow \quad v = \frac{34}{8} = \boxed{\frac{17}{4}}$$

7) (10 points) Compute the value of the matrix game below.

$$\begin{bmatrix} 3 & 2 & 6 & 2 \\ 5 & 3 & 3 & 4 \\ 0 & -1 & 7 & -2 \\ 0 & 0 & -1 & -8 \end{bmatrix}$$

First look for saddle pts (see ∇, Δ above)

There is saddle pt (marked ✡).

So $\underline{m} = \overline{m} = 3$

This implies $\boxed{v = 3}$

8) (10 points) Consider a bimatrix game for which the expected payoff to player i is $\Pi_i(p, q)$, $i = 1, 2$, when the players choose mixed strategies p and q, respectively. Use the notation P and Q for the sets of mixed strategies for players I and II, respectively.

a) Provide the mathematical definitions of the best response correspondences $R_1(q)$, $q \in Q$ and $R_2(p)$, $p \in P$.

b) Provide the mathematical definition of Nash equilibrium for this game, in terms of the best response correspondences $R_1(q)$ and $R_2(p)$.

a) $R_1(q) = \{ p \in P : \Pi_1(p, q) \geq \Pi_1(p', q)$ for all $p' \in P \}$.

$R_2(p) = \{ q \in Q : \Pi_2(p, q) \geq \Pi_2(p, q')$ for all $q' \in Q \}$.

b) (p, q) is a Nash equilibrium if

$p \in R_1(q)$ and $q \in R_2(p)$.