Math 31A Section 2B Week 0

Logistics

TA: Raymond Chu

Office hours: 11AM - 12PM Tuesday Or by appointment

Office hours are an hour where students can come to my office and ask any questions about the course or math

Email: rchu@math.ucla.edu - Include "[31A]" in email subject : will respond in 48 hours

Website: www.math.ucla.edu/~rchu

Anonymous feedback + suggestions at my website.

Will use discussion to quickly review topics that are useful for the week's subject then have students work on worksheet in breakout rooms of 4-5 people while I circulate throughout the rooms helping students.

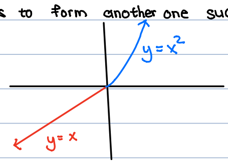
Tips to do well in college math Read textbook before lecture! Understand the book's examples & do extra unassigned problem!!

Review: Helpful tricks and concepts for next week's topic of limits

Piecewise Function | Given two functions such as $f(x) := x$ & $g(x) := x^2$ (the symbol ":=" means defined)

we can "glue" these functions to form another one such as

$$h(x) := \begin{cases} x & \text{if } x \leq 0 \\ x^2 & \text{if } x > 0 \end{cases}$$



Graph of h(x)

i.e. to graph h we glue the graphs of f & g at x=0.

End points of piecewise functions

For

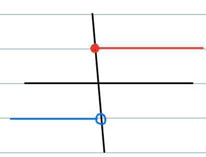
$$h(x) := \begin{cases} x & \text{if } x \leq 0 \\ x^2 & \text{if } x > 0 \end{cases} \quad \& \quad j(x) := \begin{cases} x & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$$

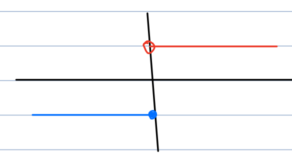
have different functions at the end point (at $x=0$ $h(x)=x$ while at $x=0$ $j(x)=x^2$)

but this doesn't matter for h & j since $f(x) := x$ & $g(x) := x^2$ have $f(0) = g(0) = 0$

so $h(x) = j(x)$ for any x.

In general, the end point matters: let $\tilde{f}(x) := \begin{cases} -2 & \text{if } x < 0 \\ 2 & \text{if } x \geq 0 \end{cases}$ and $\tilde{g}(x) := \begin{cases} -2 & \text{if } x \leq 0 \\ 2 & \text{if } x > 0 \end{cases}$



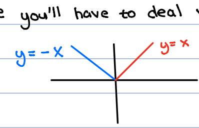


$\tilde{f}(0) = 2 \neq \tilde{g}(0) = -2$

Absolute Value : There's a good chance you'll have to deal w/ absolute value inequalities

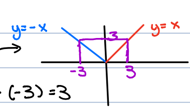
We let

$$f(x) := |x| := \begin{cases} -x & \text{if } x \leq 0 \\ x & \text{if } x > 0 \end{cases}$$



Example) Find x such that $|x| \leq 3$

Geometrically we want the x that live in the purple box



But as $|3| = 3$ & $|-3| = -(-3) = 3$

this is the region $-3 \leq x \leq 3$

In general the region $|x| \leq a$ is x such that $-a \leq x \leq a$

Example) Find x such that

$$|x-3| \leq 5$$

Trick: Let $y := x-3$ then it is

$$|y| \leq 5 \text{ i.e.}$$

$$-5 \leq y \leq 5$$

$$\text{i.e. } -5 \leq x-3 \leq 5$$

$$\text{i.e. } -5+3 \leq x-3+3 \leq 5+3$$

$$\text{i.e. } -2 \leq x \leq 8$$

Factoring Sometimes we can simplify rational functions by factoring

Example) $f(x) := \frac{x^2-16}{x-4}$

Note $(x-4)(x+4) = x^2 + 4x - 4x - 16 = x^2 - 16$

so $\frac{x^2-16}{x-4} = \frac{(x-4)(x+4)}{(x-4)} = \cancel{\frac{(x-4)}{(x-4)}}(x+4) = x+4$

↑ will be justified next week w/ limits

$f(x) = x+4$ which is a lot simpler than $\frac{x^2-16}{x-4}$

This simplification can help solve alot of limit problems

Rationalizing the Denominator Can also be used to simplify expressions with $\sqrt{x}$ on numerator or denominator

Example) $f(x) := \frac{\sqrt{x}-1}{x-1}$

Note if we have the expression $\sqrt{x}+a$ then we have $(\sqrt{x}+a)(\sqrt{x}-a) = x - a\sqrt{x} + a\sqrt{x} - a^2 = x - a^2$

Taking $a = -1$ gives $(\sqrt{x}-1)(\sqrt{x}+1) = x - (1)^2 = x - 1$ (*)

The trick is to multiply by $\frac{\sqrt{x}-a}{\sqrt{x}-a} = 1$ to get

$$\frac{\sqrt{x}-1}{x-1} = \frac{\sqrt{x}-1}{x-1}\underbrace{\left[\frac{(\sqrt{x}+1)}{(\sqrt{x}+1)}\right]}_{1} \overset{(*)}{=} \frac{x-1}{(x-1)(\sqrt{x}+1)} = \frac{\cancel{x-1}}{(\cancel{x-1})(\sqrt{x}+1)} = \frac{1}{\sqrt{x}+1}$$

This trick will be useful for limit problems