Logistics

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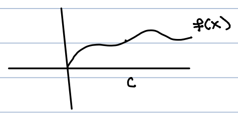
Read by yourself

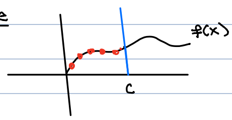
Limit: Given a function $f(x)$ and a number c



We say that the limit as x approaches from below c for f is the value $f(x)$ approaches for $\boxed{x<c}$ as x gets closer & closer to c while x is strictly less than c

denoted as $\lim_{x\to c^-} f(x)$ [also called left limit]

Picture



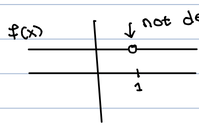
$\lim_{x\to c^-} f(x)$ is what the red dots approach when it gets closer & closer to the blue line

Note the red dots x coordinate are strictly less than c

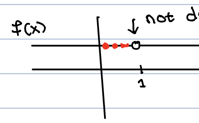
This is important since f may not be defined at c but its limit can be!

For example $f(x) = \frac{x-1}{x-1}$ is not defined at $x=1$ (since plugging in 0 gives 0/0 which is indeterminate)

but for any $x \neq 1$ we have $f(x) = 1$



but



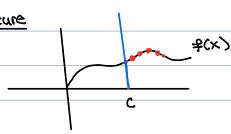
but • approaches 1

so $\lim_{x\to 1^-} f(x) = 1$ but $f(1)$ is indeterminate

The right limit at c for $f(x)$ is the value $f(x)$ approaches for $\boxed{x>c}$ as x gets closer & closer to c while x is strictly greater than c

This is written $\lim_{x\to c^+} f(x)$

Picture



$\lim_{x\to c^+} f(x)$ is what the red dots approach when it gets closer & closer to the blue line

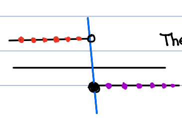
We say the limit of f as x approaches c is L if

$\lim_{x\to c^+} f(x) = \lim_{x\to c^-} f(x) = L$

and write $\lim_{x\to c} f(x) = L$

Examples (Left limit may not equal right limit and may not equal $f(c)$)

$$f(x) := \begin{cases} 1 & \text{if } x<0 \\ -1 & \text{if } x \geq 0 \end{cases}$$



The red dots approach 1 as they approach the blue line $(x=0)$: i.e. $\lim_{x\to 0^-} f(x) = 1$

The purple dots approach -1 as they approach the blue line $(x=0)$: i.e. $\lim_{x\to 0^+} f(x) = -1$

And $f(0) = 1$ but $\lim_{x\to 0^+} f(x) = -1 \neq 1 = \lim_{x\to 0^-} f(x)$ & $\lim_{x\to 0^-} f(x) \neq f(0)$

[redacted]