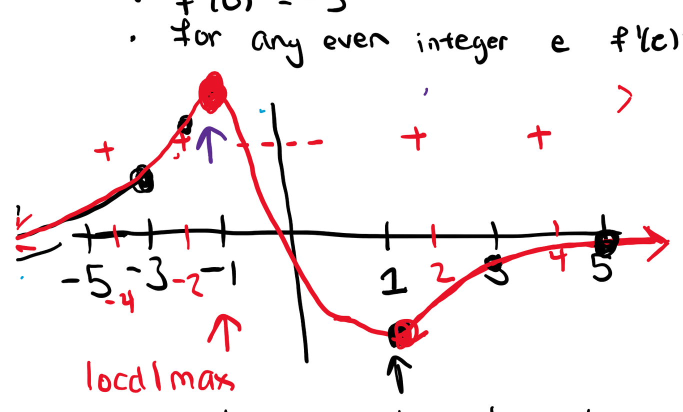
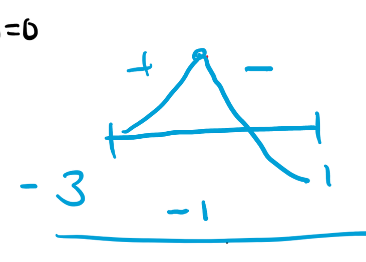
Thursday, December 3, 2020 8:51 AM

#3 on the midterm

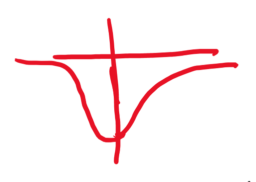
f is cts & f' is cts such that

- the only crit pts are at odd integers
- $f'(0) = -5$
- for any even integer c $f'(c)=0$





$-\frac{1}{1+x^2}$



a) Where can there be a local max?
By the critical pt thm $\Rightarrow$ has to be an odd integer
$\boxed{x=-1}$

b) Where can there be a local min?
$\boxed{x=1}$

c) Can there be an absolute max?
Yes (If there is a horizontal asymtote thats smaller $f(-1)$ then $f(-1)$ becomes a global max)

d) Is there a local max on $(-8,-3)$?
No!
B/c the function is strictly ~~the~~ max is at $x=-3$ but this is not in the interval

e) ~~Can there be~~ a local max on $[-3,5]$
$\boxed{x=-1,\ x=5}$

Sapling 17)

SIMPLIFYING

$$\int \frac{14x^3+18x-40}{x^2} = \int 14x + \frac{18}{x} - \frac{40}{x^2}$$

sum law
$$= \left(\int 14x\right) + \left(\int \frac{18}{x}\right) + \left(\int -\frac{40}{x^2}\right)$$

constants factor out
$$= 14\int x + 18\int \frac{1}{x} - 40\int \frac{1}{x^2}$$

Power law
$$= 14x^2/2 + 18\int\frac{1}{x} + 40/x + \underline{\underline{C}}$$

Remember this Easy way to lose pts

$\int F'(x) = F(x) + C$

$\frac{d}{dx}(\ln|x|) = \frac{1}{x}$

$\int \frac{1}{x} = \int \frac{d}{dx}(\ln|x|) = \ln|x| + C$

$= 14x^2/2 + 18\ln|x| + 40/x + C$

Lemma) (Switching Index)

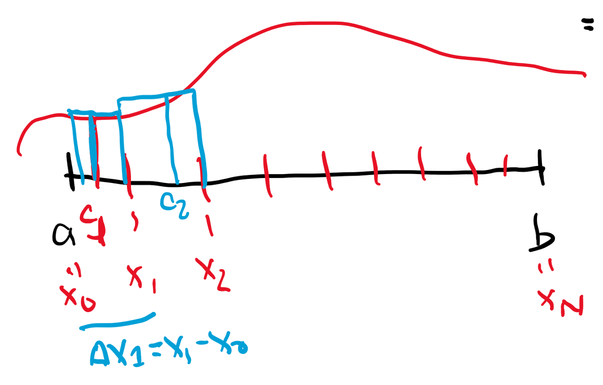
$$\sum_{j=0}^{N-1} f(x_j) = \sum_{j=1}^{N} f(x_{j-1})$$

Proof)
$$\sum_{j=0}^{N-1} f(x_j) = f(x_0) + f(x_1) + \dots + f(x_{N-1})$$
$$\sum_{j=1}^{N} f(x_{j-1}) = f(x_0) + f(x_1) + \dots + f(x_{N-1})$$

7) Use R_N approximation to find area under graph of $f(x) = 23\sin(x)$ on $[0,\pi]$

Soln) Area under graph $= \int_0^\pi 23\sin(x)$

$$= \lim_{N\to\infty}\sum_{j=0}^{N-1} \Delta x_j f(c_j)$$



R_N means right end pt w/ uniform step size

$[a,b] = [x_0, x_1] \cup [x_1, x_2] \cup \dots \cup [x_{N-1}, x_N]$

$\Delta x = \frac{b-a}{N}$

$[x_0, x_0+\Delta x] \cup [x_0+\Delta x, x_0+2\Delta x] \cup \dots \cup [(N-1)\Delta x + x_0, x_0+N\Delta x]$

$x_0 \quad x_1 \quad x_2 \quad x_3$

$x_j = x_0 + j\Delta x$

$R_N \Rightarrow$ Want as $\lim_{N\to\infty}\sum_{j=0}^{N-1} \Delta x f(x_{j+1})$

$\lim_{N\to\infty}[\Delta x f(x_1) + \Delta x f(x_2) + \dots + \Delta x f(x_N)]$

$$= \lim_{N\to\infty}\sum_{j=0}^{N-1}\left(\frac{\pi-0}{N}\right) f((j+1)\Delta x)$$

$f(x) = 23\sin(x)$

$$= \lim_{N\to\infty}\sum_{j=0}^{N-1}\frac{\pi}{N}\, 23\sin\left(\frac{\pi(j+1)}{N}\right)$$

$$= \lim_{N\to\infty}\sum_{j=1}^{N}\pi/N\; 23\sin(\pi j/N)$$

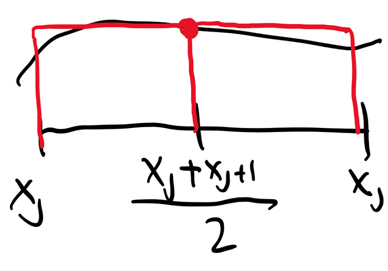
8) Use M_N approximation to approx area under graph of $\tan(x)$ over $[0.3, 0.7]$

$$\int_{0.3}^{0.7}\tan(x) = \lim_{N\to\infty}\sum_{j=0}^{N-1}\Delta x_j f(c_j) \qquad c_j = \frac{x_j+x_{j+1}}{2}$$

$[x_j, x_{j+1}]$

$\frac{x_{j+1}+x_j}{2}$ ← mid pt

Use middle of each interval $\left(\frac{x_j+x_{j+1}}{2}\right)$ as height of rectangle



Want $\Delta x f\left(\frac{x_j+x_{j+1}}{2}\right)$

$\Delta x = \frac{b-a}{N}$ because its uniform
$b=.7,\ a=.3$

$\Delta x = .4/N$, $x_j = a + j\Delta x = .3 + j(.4/N)$

$$\frac{x_j+x_{j+1}}{2} = \frac{.3 + j(.4/N) + .3 + (j+1)(.4/N)}{2}$$
$$= \frac{.3+.3 + j(.4/N) + (j+1)(.4/N)}{2}$$
$$= \frac{.6 + (2j+1)(.4/N)}{2}$$
$$= .3 + (j+1/2)(.4/N)$$

$\Delta x = .4/N$, $\frac{x_j+x_{j+1}}{2} = .3 + (j+1/2)(.4/N)$

$$\lim_{N\to\infty}\sum_{j=0}^{N-1}\Delta x f\left(\frac{x_j+x_{j+1}}{2}\right) = \lim_{N\to\infty}\sum_{j=0}^{N-1}\left(\frac{.4}{N}\right)\tan\left(.3 + (j+1/2)(.4/N)\right)$$
$$= \lim_{N\to\infty}\sum_{j=1}^{N}\left(\frac{.4}{N}\right)\tan\left(.3 + (j-1+1/2)(.4/N)\right)$$
$$= \lim_{N\to\infty}\sum_{j=1}^{N}(.4/N)\tan\left(.3 + (j-1/2)(.4/N)\right)$$

5.1.73) Find
$$\lim_{N\to\infty}\sum_{j=1}^{N}\frac{1}{N}\sqrt{1-(j/N)^2}$$

→ 1) Can I rewrite this as a Riemann Sum to rewrite the limit as an integral we can hopefully solve?

Soln) Step 1) Rewrite as a riemann sum

Riemann Sum in general looks like $\sum_{j=1}^{N}\Delta x_j f(c_j)$

$\sum_{j=1}^{N}\frac{1}{N}\sqrt{1-(j/N)^2}$ ← $f(c_j)$

Make guesses: $\Delta x_j = 1/N$
If we make this guess then $f(c_j) = \sqrt{1-(j/N)^2}$ looks like $\sqrt{1-x^2}$

Make the guess $c_j = j/N$
to get $\sqrt{1-c_j^2} = f(c_j)$
$= \sqrt{1-(j/N)^2}$

$f(x) = \sqrt{1-x^2}$

→ Riemann Sum of $\sqrt{1-x^2}$ on $[0,1]$

As $j\to\infty$ $c_N \to b$, $c_1 \to a$

$a = c_j$, b; $x_0\ x_1\ x_2\ x_N$

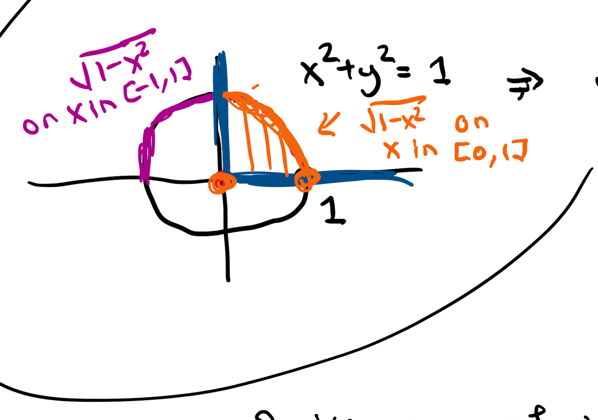
$c_N = 1 \qquad c_j = j/N$

$c_1 = 1/N \xrightarrow[N\to\infty]{} 0$

$$\boxed{\int_0^1 \sqrt{1-x^2}\,dx}$$

Solving the integral

$\int_0^1\sqrt{1-x^2}\,dx$
$=$
Area under this graph ($f(x) = \sqrt{1-x^2}$) on $[0,1]$



$x^2+y^2 = 1 \Rightarrow y^2 = 1-x^2$
$y = \pm\sqrt{1-x^2}$

the plus corresponds to half circle

So its area of circle of radius divided by 4
$= \pi/4$
area ~~of~~ under graph of $\sqrt{1-x^2}$ on $[0,1]$

$$\int_0^1\sqrt{1-x^2}\,dx = \pi/4$$

$\Delta x_1 = 6-4 = 2 \quad \Delta x_2 = 9-6 = 3 \quad \Delta x_3 = 13-9 = 4$

$[4,13] = [4,6] \cup [6,9] \cup [9,13]$

$C = \{4, 7.5, 11.5\}$

$c_1 \quad c_2 \quad c_3$

$$\sum_{i=1}^{3}\Delta x_i f(c_i)$$

$2f(4) + 3f(7.5) + 4f(11.5)$