Thursday, December 10, 2020 8:43 AM

OH this week
1PM - 2PM today
+ 9 AM - 10AM Sat
+ 2PM - 3PM Sat

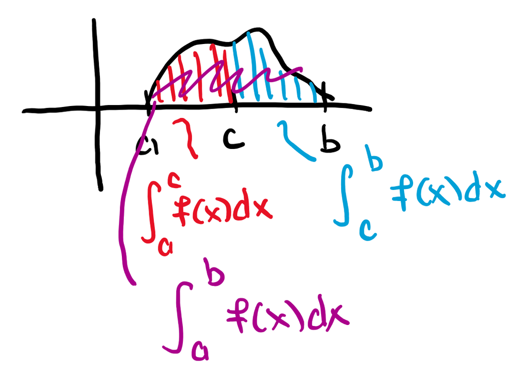
SMC Hours
11AM-12PM today

You have a final this Sunday

Ben is hosting a review session Saturday 3PM - 5PM.

Please fill out class evaluations

Theorem | If $a<c<b$ then $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$



• Mainly useful for piecewise functions

#3 Sapling | If $f(x) = \begin{cases} 1-x^3 & \text{if } x \le 1 \\ x^2 & \text{if } x > 1 \end{cases}$

Find $\int_{-1}^2 f(x)\,dx = \int_{-1}^1 f(x)dx + \int_1^2 f(x)dx$

$= \int_{-1}^1 (1-x^3)\,dx + \int_1^2 x^2\,dx$

$= (x - x^4/4)\Big|_{-1}^{1} + (x^3/3)\Big|_1^2$

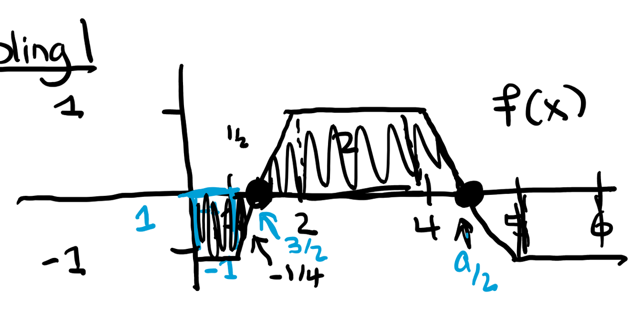
$= (1 - 1^4/4 - (-1 - (-1)^4/4)) + (2^3/3 - 1/3)$

$= \boxed{13/3}$

Also $\int_a^b f(x)dx = -\int_b^a f(x)dx$

$\int_{-1}^2 f(x) = 13/3$, $\int_2^{-1} f(x)dx = -13/3$

#14 Sapling |



(FTC 2 $A'(x) = f(x)$)
$f(x)=0$ is crit pts

$A(x) = \int_0^x f(t)dt$ ← (Signed area of the graph)

a) Whats the min of A on [0,6]

$\int_0^1 f(x)dx = -1$ $\int_{3/2}^2 f(x)dx = 1/4$ $\int_4^{9/2} f(x)\,dx = 1/4$ $\int_5^6 f(x)dx = -1$

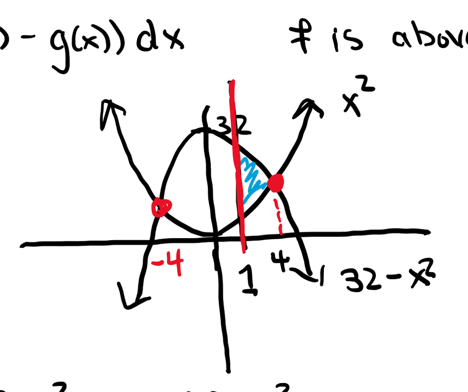
$\int_1^{3/2} f(x)\,dx = -1/4$ $\int_2^4 f(x)dx = 2$ $\int_{9/2}^5 f(x)\,dx = -1/4$

⇒ Min is $\int_0^{3/2} f(x)dx = \int_0^1 f(x)dx + \int_1^{3/2} f(x)dx = -1 - 1/4 = -5/4$

b) Asks for max of A which $\int_0^{9/2} f(x)dx = \int_0^1 f(x)dx + \int_1^{3/2} f(x)dx + \int_{3/2}^2 f(x)dx + \int_2^4 f(x)dx + \int_4^{9/2} f(x)\,dx = 5/4$

#30 Sapling | Use shell method to find volume of the solid enclosed by the graphs of $y=x^2$, $y=32-x^2$, $x=1$ rotated about y-axis

⇒ $2\pi\int_a^b x(f(x)-g(x))\,dx$ f is above g



$f(x) = 32-x^2$ (on top), $g(x) = x^2$ (on bottom)

$V = 2\pi\int_a^b x(32-x^2-x^2)$ $a=1$

b is when the graphs touch ⇒ $32-x^2 = x^2$
$32 = 2x^2$
$16 = x^2$
$x = \pm 4$
$b=4$

$2\pi\int_1^4 x(32-2x^2)dx = 2\pi\int_1^4 32x - 2x^3\,dx = 2\pi\,(16x^2 - x^4/2)\Big|_1^4 = 225\pi$

FTC II

$A(x) := \int_a^x f(t)\,dt$

(FTC II) says $A'(x) = f(x)$

Ex)
$B(x) = \int_4^x (t+3)\,dt \Rightarrow B'(x) = x+3$

$A(x) = \int_4^{x^2} (t+3)\,dt$ ←

Let $F(x) = \int_4^x (t+3)\,dt$ $F'(z) = z+3$

$F(x^2) = \int_4^{x^2} (t+3)\,dt = A(x)$

$\frac{d}{dx}(F(x^2)) = F'(x^2)\,(2x)$
$= (x^2+3)(2x)$

$\frac{d}{dx}\int_a^{g(x)} f(t)dt = f(g(x))\cdot g'(x)$

Proof
$A(x) = \int_a^x f(t)\,dt$, $A(g(x)) = F(x)$

$\frac{d}{dx}(A(g(x)) = A'(g(x))\,g'(x)$
$\underset{\text{FTC 2}}{=} f(g(x))\,g'(x)$

Ex 6 Worksheet
$\frac{d}{dx}\int_1^{\sin(x^2+1)} \tan(\sin(t))\,dt$

2 ways) $\frac{d}{dx}\int_a^{g(x)} f(t)dt = f(g(x))\,g'(x)$ OR

$A(x) = \int_1^x \tan(\sin(t))$ then $A(\sin(x^2+1)) = \int_1^{\sin(x^2+1)} \tan(\sin(t))\,dt$

$d/dx\,A(\sin(x^2+1)) = \underbrace{\tan(\sin(\sin(x^2+1)))}_{f(g(x))} \cdot \cos(x^2+1)\cdot 2x$

7) $\frac{d}{dx}\int_1^{(\sin(x)+10)^{2020}} \sin\left(1 + \frac{1}{t^2+1} + t^2\right)dt = d/dx\,F(x)$

$A(x) = \int_1^x \sin\left(1 + \frac{1}{t^2+1} + t^2\right)dt$ $\boxed{A((\sin(x)+10)^{2020}) = F(x)}$

⇒ $F'(x) = A'((\sin(x)+10)^{2020})\ \boxed{2020\,(\sin(x)+10)^{2019}\cos(x)}$

$A'(z) = \sin\left(1 + \frac{1}{1+z^2} + z^2\right)$

15) Suppose f is a differentiable function and $A(x) = \int_a^x f(t)\,dt$

1) A is decreasing ⇒ what

$d/dx\,A(x) = f(x) \Rightarrow f(x) \le 0$ (f lies below the x-axis)

$f \le 0$ - - - -
$f \ge 0$ + + +

2) If A has a min (at $x=c$) ⇒ crit pt ⇒ $A'(x) = 0 = f(x)$
(crosses x-axis from negative to positive)

If g is a diff & decreasing function then $g' \le 0$

3) A is concave up ⇒ $A'' \ge 0$
$f' \ge 0$ (or f is increasing)

Integration by Sub

Ex 1) $\int x\,(8x+9)^{12}\,dx$

$\int f(g(x))\,h(x)\,dx$

→ Usually choose $u = g(x)$

$f = x^{12}$, $g = 8x+9$
$u = 8x+9 \Rightarrow du = 8dx$ ($du/8 = dx$)

$\int \frac{x}{8}(u)^{12}du = \int\left(\frac{u-9}{8}\right)\frac{1}{8}(u)^{12}\,du = \frac{1}{64}\int(u-9)\,u^{12}\,du$

$\frac{u-9}{8} = x$

$= \frac{1}{64}\int u^{13} - 9u^{12}\,du$

$= \frac{1}{64}\left(\frac{u^{14}}{14} - 9\frac{u^{13}}{13}\right) + C$

$= \frac{1}{64}\left(\frac{(8x+9)^{14}}{14} - \frac{9}{13}(8x+9)^{13}\right) + C$

easy way to lose points

(Definite int $\int_a^b$, indefinite $\int$ — always add a $+C$)

$\int x^{10}\sin(x^{11}+1)\,dx$ $u = x^{11}+1$, $du = 11x^{10}\,dx$

$= \int \frac{x^{10}}{11x^{10}}\sin(u)\,du = \frac{1}{11}\int\sin(u)\,du = -\frac{1}{11}\cos(u) + C$
$= -1/11\cos(x^{11}+1) + C$

$\int 2x\sin(x^2+2020)\,dx$ $u = x^2+2020$, $du = 2x\,dx$

$= \int \frac{2x}{2x}\sin(u)\,du = \int\sin(u)\,du = -\cos(x^2+2020) + C$

16) $\int_1^2 \frac{(x^2+1)}{(x^3+3x)^2}\,dx$ $u = x^3+3x$ $du = 3x^2+3\,dx$
$= 3(x^2+1)\,dx$

$= \int_{1^3+3}^{2^3+3(2)} \frac{(x^2+1)}{3(x^2+1)}\frac{1}{u^2}\,du = \int_4^{14}\frac{1}{3u^2}\,du$

$= -1/3u\Big|_4^{14}$

$= -1/3\left(\frac{1}{14} - 1/4\right) = 5/84$