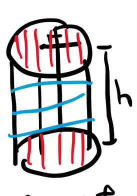
Tuesday, December 1, 2020 8:43 AM

4.6.46) Find the dimensions of a cylinder of volume 1m s.t. it has minimal cost if the material on the top and bottom of the cylinder cost 3 times as much as the sides

Soln|



$V = 1 = \pi r^2 h \Rightarrow \boxed{h = \frac{1}{\pi r^2}}$

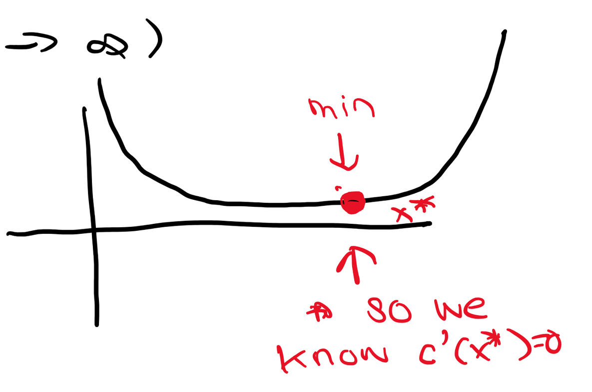
Find area of top, bottom, & side parts.
Area of top & bottom are πr^2 each
On the sides the area is $2\pi r h$
If c is the total cost & s is the cost of the material on the sides then

$c = \underbrace{3s(\pi r^2) + 3s(\pi r^2)}_{\text{amount of mat on top \& bottom multiplied by 3s}} + \underbrace{s}_{\text{cost of material on sides}}\ \underbrace{(2\pi r h)}_{\text{amount of mat on sides}} \leftarrow$ 2 variables

$c = 3s(\pi r^2) + 3s(\pi r^2) + s(2\pi r(\frac{1}{\pi r^2}))$

$\boxed{c = 6s(\pi r^2) + s(2/r)}$ (we know that as $r \to \infty$ $c \to \infty$)

[really small number, really big number]



min

x^*

so we know $c'(x^*) = 0$

$\frac{dc}{dr} = 12s\pi r - 2s/r^2$

$0 = 12s\pi r - 2s/r^2 \Rightarrow 0 = 12\pi r^3 - 2$

multiplied by r^2

$2 = 12\pi r^3$

$1/6\pi = r^3$

$\boxed{r = \sqrt[3]{1/6\pi}}$

So then as $1 = V = \pi r^2 h$

$\frac{1}{\pi r^2} = h \Rightarrow \boxed{h = \frac{1}{\pi(1/6\pi)^{2/3}}}$

Sapling 17)

$\int \frac{14x^3 + 18x - 40}{x^2} = \int 14x + 18x^{-1} - 40x^{-2}$

sum law

$= \left(\int 14x\right) + \left(\int 18x^{-1}\right) + \left(\int -40x^{-2}\right)$

$\boxed{\int x^{-1} = \ln(x)}$

constant factor out

$= 14\int x + 18\int x^{-1} - 40\int x^{-2}$

power law

$= 14\frac{(x^{1+1})}{1+1} + 18\ln(x) - 40\frac{x^{-2+1}}{-2+1}$

$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$

$= \frac{14x^2}{2} + 18\ln(x) + 40x^{-1} + C$

$\int \frac{d}{dx}(f(x)) = f(x) + \underline{\underline{C}}$

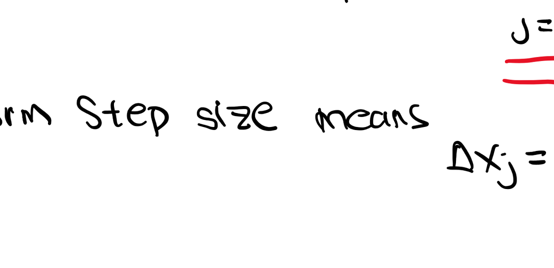
Lemma|

$\sum_{j=0}^{N-1} f(x_j) = \sum_{j=1}^{N} f(x_{j-1}) = f(x_0) + f(x_1) + \dots + f(x_{N-1})$

$\| \quad f(x_0) + f(x_1) + \dots + f(x_{N-1})$

Sapling 7) Use R_N approximation to find Area under graph of $f(x) = 23\sin(x)$ on $[0, \pi]$.

Soln| Want to write $[0,\pi] = [x_0, x_1] \cup [x_1, x_2] \cup \dots \cup [x_{N-1}, x_N]$



Want to write area as Riemann Sum

$\lim_{N\to\infty} \sum_{j=0}^{N-1} \Delta x_j f(x_{j+1})$

right end pt is x_{j+1}

Uniform Step size means

$\Delta x_j = \frac{b-a}{N}$

$(N = 2)$ $\Delta x = \frac{1-0}{2} = \frac{1}{2}$

0 1/2 1

Evenly cuts out the interval $[a,b]$ into N pieces

$\Delta x = \frac{\pi - 0}{N} = \frac{\pi}{N}$

$[0, \pi/N] \cup [\pi/N, \pi/N + \pi/N] \cup [2\pi/N, 3\pi/N]$ ....

$x_j = a + j\Delta x$ $\quad x_0 = a$ $\quad \Rightarrow x_{j+1} = a + (j+1)\Delta x$

start out at a
& each interval add Δx
Be careful of index on sum (this is true if I start at $j=0$)

[But if j starts at 1 then
$x_j = a + (j-1)\Delta x$]
$x_1 = a + (1-1)\Delta x = a$

$\lim_{N\to\infty} \sum_{j=0}^{N-1} \Delta x_j f(x_{j+1}) = \lim_{N\to\infty} \sum_{j=0}^{N-1} \frac{\pi}{N} f((j+1)\Delta x)$

$= \lim_{N\to\infty} \sum_{j=0}^{N-1} \pi/N\,(23\sin((j+1)\Delta x))$

$= \lim_{N\to\infty} \sum_{j=1}^{N} \pi/N\,(23\sin((j)\Delta x))$

8) Use M_N approximation to approx area under the curve of $\tan(x)$ over $[0.3, 0.7]$

M_N = middle approximation $\quad [x_j, x_{j+1}]$

$\frac{x_j + x_{j+1}}{2} \leftarrow$ middle of the interval

$\lim_{N\to\infty} \sum_{j=0}^{N-1} \Delta x_j f\left(\frac{x_j + x_{j+1}}{2}\right)$

only true when we start at $j=0$

$x_j = a + j\Delta x$

$\Delta x = \frac{b-a}{N} = \frac{0.7-0.3}{N} = \frac{0.4}{N}$

$\frac{x_j + x_{j+1}}{2} = \frac{0.3 + j\Delta x + 0.3 + (j+1)\Delta x}{2}$

$= \frac{0.6 + (2j+1)\Delta x}{2}$

$= 0.3 + (j + \frac{1}{2})\Delta x$

$\lim_{N\to\infty} \sum_{j=0}^{N-1} \left(\frac{0.4}{N}\right) \tan(0.3 + (j + 1/2)\Delta x)$

$= \lim_{N\to\infty} \sum_{j=1}^{N} \left(\frac{0.4}{N}\right) \tan(0.3 + (j - 1 + 1/2)\Delta x)$

$= \lim_{N\to\infty} \sum_{j=1}^{N} \left(\frac{0.4}{N}\right) \tan(0.3 + (j - 1/2)\Delta x))$

5.1.73

Find $\lim_{N\to\infty} \sum_{j=1}^{N} \frac{1}{N} \sqrt{1 - (j/N)^2} \leftarrow$

Soln) We are going to write this as a Riemann Sum to get it into an integral.

Riemann Sum looks like

$\sum_{j=1}^{N} \Delta x_j f(c_j)$ $\quad c_j$ in $[x_j, x_{j+1}]$

a b

$\sqrt{1 - (j/N)^2}$

Lets guess $\Delta x_j = 1/N$

$\to$ Want $f(c_j) = \sqrt{1 - (j/N)^2}$ $\quad \boxed{\text{Guess } c_j = j/N}$

$\to f(j/N) = \sqrt{1 - (j/N)^2}$

$\boxed{f(x) = \sqrt{1 - x^2}}$ $\quad (x = j/N)$

$c_j = j/N$, $\Delta x = 1/N$

$\lim_{N\to\infty} \sum_{j=1}^{N} \frac{1}{N} \sqrt{1 - (j/N)^2} = \lim_{N\to\infty} \sum_{j=1}^{N} \Delta x\, f(j/N)$

$c_j = j/N$

As $N \to \infty$ $\quad c_N \to b \quad c_N = N/N = 1$
$c_1 \to a \quad c_1 = 1/N \to 0$

c_1 b

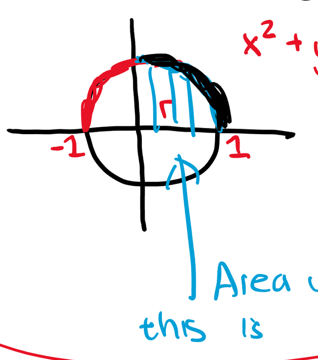
0 1/N 2/N 3/N ... 1

Riemann sum of $\sqrt{1-x^2}$ on $[0,1]$

$= \int_0^1 \sqrt{1 - x^2}\, dx$

$\|$

Area under the graph of $\sqrt{1-x^2}$ from $x=0$ to 1



$x^2 + y^2 = r^2 \leftarrow$ eqn of a circle
guess $r=1$
$x^2 + y^2 = 1$

$y^2 = 1 - x^2$
$y = \pm\sqrt{1 - x^2}$

Area under this is

$y = \sqrt{1 - x^2}$ $\quad$ x to be in $[0,1]$

$\int_0^1 \sqrt{1 - x^2}$

Area of a quarter is $\pi r^2/4$ $(r=1)$ $= \pi/4$

$\int_0^1 \sqrt{1 - x^2}\, dx = \pi/4$