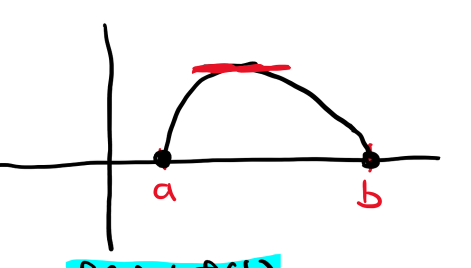
Tuesday, November 17, 2020 8:50 AM

Office hours: 1PM-2PM

Goals| Rolle vs IVT
Related rates
Hw #72B
Extrema

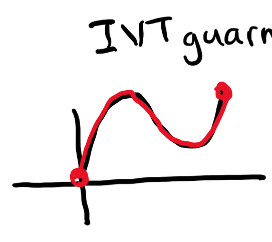
Rolle's Thm| Says if $f(x)$ is continous on $[a,b]$ and differentiable on (a,b) ~~θ~~ with $f(b)=f(a)$ then there is a c in (a,b) s.t. $f'(c)=0$



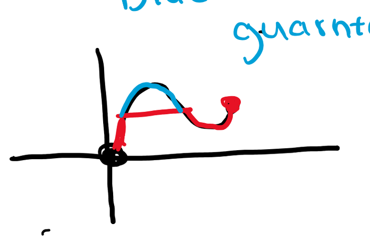
Useful to show $f(a) \neq f(b)$. Because if $f(a)=f(b)$ then we can find a c in (a,b) s.t. $f'(c)=0$. ←
↑ WTS this can never happen

Mostly done by showing that $f'>0$ or $f'<0$ on (a,b)

IVT guarntees red



blue IVT doesn't guarntee to exist



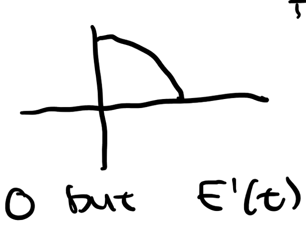
(Only continuity)
IVT| Mainly useful for showing existence of values ←

Practice Midterm #3| Calvin expects $E(t)=a(t^3+bt)+c\sin(t)$ for some $a,b,c>0$ But through experiments he has found $E(.335)=E(.92)$? Why does this contradict his theory?

Soln) If $E(.335)=E(.92)$ then Rolle's thm applies since this function is diff on $(.335,.92)$ & continuous on $[.335,.92]$ to say there is a c between .335 & .92 s.t.
$E'(c)=0$

But $E'(t)=a(3t^2+b)+c\cos(t)$ and on $[.335,.92]$
$E'(t)>0$ because $a,b,c>0$

$\pi/2$ $\cos(x)>0$ on $(0,\pi/2)$



So contradiction since $E'(c)=0$ but $E'(t)>0$ on $[.335,.92]$

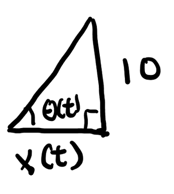
Soln 2| Recall if $E'(t)>0$ on (a,b) then $E(t)$ is increasing on (a,b)
So as $E'(t)>0$ on $(.335,.92)$ then we have its increasing on this interval. which implies
$E'(.92) > E'(.335)$ but we are told
$E'(.92) = E'(.335)$, so Calvin is wrong

Ex) $f(x)=x^{2020}+10x^6+20x$ show it has no zeros on $(0,\infty)$

Soln| Note $f(0)=0^{2020}+10(0)^6+20(0)=0$
$f'(x)=2020x^{2019}+60x^5+20>0$ on $(0,\infty)$
That f is increasing on $(0,\infty)$ so for any $x>0$
$f(x)>f(0)=0$

If there was another 0 say at c ~~ϵf~~ in $(0,\infty)$ then $f(c)=f(0)=0$ and f is diff & cts on $[0,c]$ so rolle's there is a d in $(0,c)$ such that
$f'(d)=0$
but $f'(x)>0$ ↑

Related Rates|



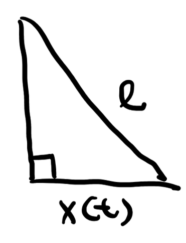
Given $dx/dt=5$ Find $d\theta/dt$ when $x=20$
$\tan(\theta)=\frac{10}{x(t)} \Rightarrow x(t)\tan(\theta)=10$

So diff to get
$x'(t)\tan(\theta)+x(t)\frac{d}{dt}(\tan(\theta))=0$

$d/dx \tan(x)=\sec^2(x)$
$\tan(x)=\frac{\sin(x)}{\cos(x)}$ ← quotient rule

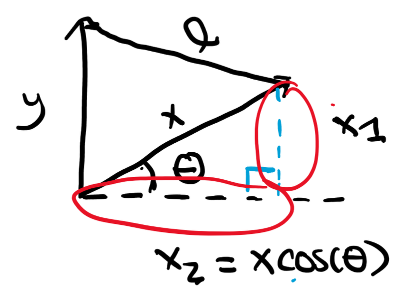
$x'(t)\tan(\theta)+x(t)\sec^2(\theta)\,d\theta/dt=0$
$5\tan(\theta)+20\sec^2(\theta)\,d\theta/dt=0$ ←
Know
$\tan(\theta)=10/20=1/2 \to \theta=\arctan(1/2)$

Ex 2)

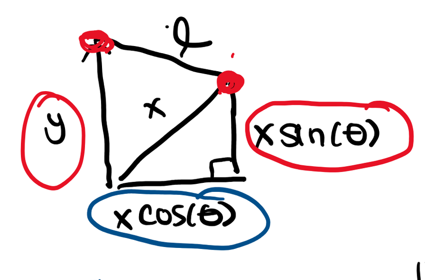


$dx/dt=10$ Find $d\ell/dt$ when $x=2$
$\ell^2=2^2+(x(t))^2$
$2\ell\ell'=2(x(t))x'(t)$ ←
$\ell=\sqrt{2^2+(x(t))^2}$ $x(t)=2$ $dx/dt=10$

Ex3) Practice midterm #3
Want $d\ell/dt$ at $x=y=10$
Given $dx/dt=3$ $dy/dt=2$, θ



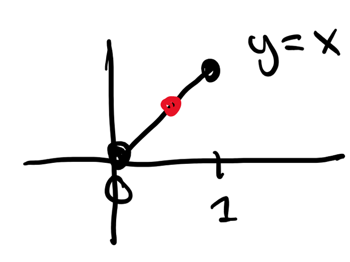
$x_1=x\sin(\theta)$
$\sin(\theta)=x_1/x$
$x_2=x\cos(\theta)$



ℓ = distance between the 2 points
$d((x,y),(u,v))=\sqrt{(x-u)^2+(y-v)^2}$
$\ell=d((x\cos\theta,x\sin\theta),(0,y))$
$\boxed{\ell^2=(x\cos\theta)^2+(x\sin\theta-y)^2}$ ← have ℓ

$2\ell\,d\ell/dt=2(x\cos\theta)\,dx/dt+2(x\sin\theta-y)[dx/dt\sin\theta-dy/dt]$

If you want extrema of f on $[a,b]$
Extrema| Find critical points of $f(x)$ on $[a,b]$ i.e. $f'(x)=0$ or DNE. Let $x_1,x_2,\dots,x_n$ be the critical points
- To find global max/min, $\{f(a),f(b),f(x_1),f(x_2),\dots,f(x_n)\}$
↑ min/max of this list is the true global max/min



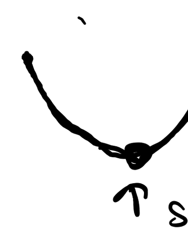
min at x^*
local means inside the interval
& for x near x^* means
$f(x^*)\le f(x)$

- To find local max/min draw a sign chart of crit pts.
(x^* is crit pt)

Ex) $f(x)=x^2-3x+2$. Find max & mins (global & local) on $[0,2]$
↑ ↑
$f'(x)=2x-3$
$2x-3=0 \Rightarrow x=3/2$

Global max & min $\{f(0), f(2), f(3/2)\}$
$f(0)=2$, $f(2)=0$, $f(3/2)=-1/4$

local max & min|
$f'(x)=2x-3$
signs of your derivative → ⊖ ⊕ $3/2$



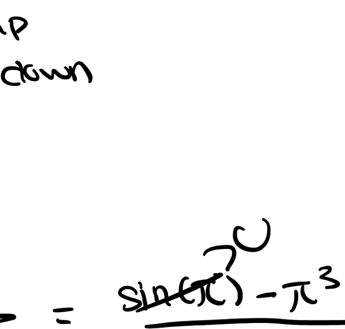
↑ so local min

$f'(3/2-1/100)=2(3/2-1/100)-3$
$=3-2/100-3=-2/100$

$f'(3/2+1/100)>0$

Hw 72b) Find the concavity of $f(x)=\tan(x)$ on $(-\pi/2,\pi/2)$
$f'(x)=\sec^2(x)$
$f''(x)=2\sec^2(x)\tan(x)$
>0 when defined

Concave up if $f''>0$
" " down if $f''<0$
inflection if $f''=0$ or DNE



$\cos(x)$ & $\sin(x)\ge 0$ on $[0,\pi/2]$
$\cos(x)\ge 0$ but $\sin(x)\le 0$ on $[-\pi/2,0]$

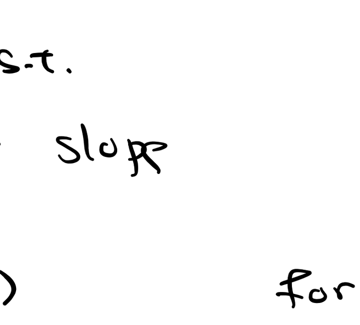
$\sec^2(x)=\frac{1}{\cos^2(x)}$ Defined $(-\pi/2,\pi/2)$

$\tan(x)>0$ on $(0,\pi/2)$ ← concave up
$\tan(x)<0$ on $(-\pi/2,0)$ ← concave down
$x=0$ ← inflection points

Worksheet
1a) average slope is on $(0,\pi)$
$\frac{f(\pi)-f(0)}{\pi}=\frac{\sin(\pi)-\pi^3-\sin(0)}{\pi}=-\pi^2$

b) $\frac{f(\pi)-f(0)}{\pi}=f'(w)$ for some w in $(0,\pi)$

$f'(x)=\cos(x)-3x^2$
$-\pi^2=\cos(w)-3w^2$
$-\pi^2+3w^2=\cos(w)$



2) $f(x)=e^x$ want to find a $c>0$ s.t.
$f'(c)=\frac{e^2-1}{2}$ ← average slope
$\frac{e^2-e^0}{2-0}\overset{MVT}{=}f'(c)$ for c in $(0,2)$
↑ $[0,2]$ of $f(x)=e^x$ average slope