Goals Chain Rule, Implicit Diff, Related Rates

Chain rule $\frac{d}{dx}(f(g(x))) = f'(g(x))\,g'(x)$

Example 1 Find $\frac{d}{dx}(\sin(\cos(x)))$

Let $f(x) = \sin(x)$ then

$f(\cos(x)) = \sin(\cos(x))$ so

let $g(x) = \cos(x)$ then

$f(g(x)) = \sin(\cos(x))$

$f'(x) = \cos(x)$, $g'(x) = -\sin(x)$

So $\frac{d}{dx}(f(g(x))) = -\cos(\cos(x))\sin(x)$

Ex 2 #33 Sapling

Let $h(x) = \sqrt{\sin(11x)\cos(11x)}$

$f(x) = \sqrt{x}$, $f(\sin(11x)\cos(11x)) = \sqrt{\sin(11x)\cos(11x)}$

$g(x) = \sin(11x)\cos(11x)$ then $f(g(x)) = h(x)$

$h'(x) = f'(g(x))\,g'(x)$

$f'(x) = \frac{1}{2\sqrt{x}}$, $g'(x) = 11\cos^2(11x) - 11\sin^2(11x)$

$$\frac{11(\cos^2(11x) - \sin^2(11x))}{2\sqrt{\sin(11x)\cos(11x)}} = \frac{11\cos(22x)}{2\sqrt{\frac{1}{2}\sin(22x)}} = \frac{11\cos(22x)}{\frac{2}{\sqrt{2}}\sqrt{\sin(22x)}} = \frac{11\cos(22x)}{\sqrt{2}\sqrt{\sin(22x)}}$$

$\cos^2(z) - \sin^2(z) = \cos(2z)$

Take $z = 11x$ $\Rightarrow$

$\sin(z)\cos(z) = \frac{1}{2}\sin(2z)$

$z = 11x$

Implicit Diff Differentiating when we have not solved for y.

Ex) Find dy/dx where $x^3 + y^2 = 1$

Solution

$\frac{d}{dx}(x^3 + y^2) = \frac{d}{dx}(1)$

$3x^2 + \frac{d}{dx}(y^2) = 0$

$3x^2 + 2yy' = 0$

$y(x)$

$f(x) = x^2$

$f(y(x)) = y^2$ ← do chain rule to get

$\frac{d}{dx}(y^2) = 2yy'$

$2yy' = -3x^2$, $y' = -\frac{3x^2}{2y}$

Ex2) Find $\frac{d}{dx}(\arctan(e^x))$

$y = \arctan(e^x)$

$\tan(y) = e^x$

$\frac{d}{dx}(\tan(y)) = \frac{d}{dx}(e^x)$

$= e^x$

Chain rule

$\frac{d}{dx}(\tan(x)) = \sec^2(x)$

$\tan(x) = \frac{\sin(x)}{\cos(x)}$ and do quotient rule

$\tan(\arctan(x)) = x$

$\tan(y) = \tan(\arctan(e^x))$

$= e^x$

$\sec^2(y)\,y' = e^x$

$y' = \frac{e^x}{\sec^2(y)} = e^x\cos^2(y) = e^x\cos^2(\arctan(e^x))$

Ex 3) Find $\frac{d}{dx}(\sin(x)^x)$ $\qquad \log(a^x) = x\log(a)$

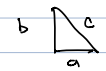
Solution $y = \sin(x)^x$

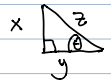
$\log(y) = \log(\sin(x)^x) = x\log(\sin(x))$

Implicit Diff & Solve

Related Rates

General Draw pictures and write out knowns & unknowns then usually use pythagorean thm $(a^2+b^2=c^2)$





$\sin(\theta) = x/z$

$\cos(\theta) = y/z$

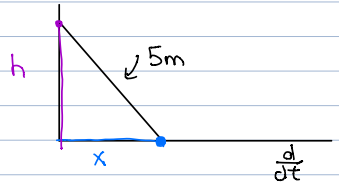
$\tan(\theta) = x/y$

Ex 1) A 5m ladder is sliding down a wall

Let h = height of the ladder at top at time t

x = distance from wall to ladder's bottom at time t

If ladder is sliding down at a rate of 1 m/s. Find dx/dt



Given $dh/dt = -1$ m/s $\qquad$ Want dx/dt

$\frac{d}{dt}(x^2+h^2) = \frac{d}{dt}(5^2)$

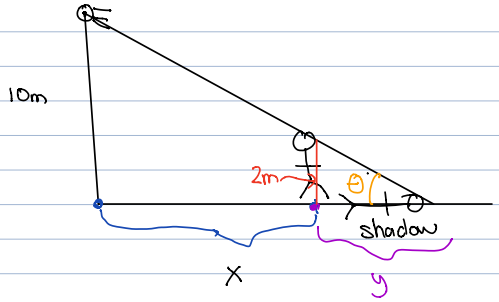
$2x\frac{dx}{dt} - 2h\frac{dh}{dt} = 0$

$2x\frac{dx}{dt} - 2h = 0$

$$\boxed{\frac{dx}{dt} = \frac{h}{x}}$$

Ex 2) A man of 2m walks away from a 10m lamp post at a speed of 1.2 m/s. Find rate at which his shadow is increasing in length.

Define variables



x is just distance of person to lamp

y is length of shadow

SOH

CAH

TOA

Given

lamp length

man's length

$dx/dt = 1.2$ m/s

Want

dy/dt

$\tan(\theta) = \frac{10}{x+y}$ $\qquad$ $\tan(\theta) = \frac{2}{y}$

$\frac{10}{x+y} = \frac{2}{y}$

$10y = 2(x+y)$

$10y = 2x+2y$ so $8y = 2x$ so $4y = x$

$$\frac{d}{dt}(4y) = \frac{d}{dt}(x)$$

$$4\frac{dy}{dt} = \frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{1}{4}\frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{1}{4}(1.2\,m/s)$$