$I^{\ddagger}$ $\lim_{x \to c} l(x) = L$ and $\lim_{x \to c} u(x) = L$ and			
there is an open interval I with a In I such that			
$L(x) \leq f(x) \leq u(x)$ for all x in I (potentially except c)	[#may be indeterminate	at x=c]	
Then $\frac{\lim_{x \to a} \pm (x)}{x} = L$			
L' + is "sandwiched" between			
$L = \int \int f(x) u(x) d f(x)$			
Ltxx			
Application1		picture a	of bound on
$(*)$ $(= \sqrt{2}) (*)$	$\lim_{x \to 0} x^2 e^{\sin(\frac{1}{x})}$		esin(1/x)
Solution	x->0 As -1 ≤ si	$\alpha(\frac{1}{2}) \leq 1$	
$\frac{26142001}{1000 k \cdot -1 \le \cos\left(\frac{5}{x}\right) \le 1}$		1(*) se 7-1	1
	⇒ e ≤ e"	sin(t) 2	
	So $x^2 e^{-1} \le x^2 e^{-1}$	since i xe	
$  s_0 -  x^3  \le x^3 \cos(\frac{5}{x}) \le  x^3 $	As 2:00 2 e' = 5	Lim xe=0	
$-30 \text{ as } \lim_{x \to 0^{-1}} - x^3  = \lim_{x \to 0^{-1}}  x^3  = 0$	Squeeze thm imp	plies $\lim_{x \to 0} x^2 e^{\sin(\frac{1}{x})} = 0$	
=> By squeeze Lim x3 cos(=)=0			
Exercise If $f_{xxx} \neq (x) = 0$ & there is a C, & Cz constants	such that	Hint: Use squeeze t	bm +
		Idea of example	
$c_{1} \leq g(x) \leq c_{2}  \text{then}  \lim_{x \to 0} \neq$	(x) = 0	The of a southe	~ /
Note both examples above are examples	o of this.		
Intermediate Value Theorem			
Let f(x) be a continuous function on [a,b] for a <b if<="" td="" then=""><td></td><td></td><td></td></b>			
\$(a) < M < \$(b) or \$(b) < M < \$(a) then there is a ce	e (a,b) such that		
\$(x)= N.	-		
	*		
$\frac{P_{\text{lcture of IVT}}}{(1)} \cdot M \text{ between } f(a) \& f(b) \\ (1) \\ (2) \\ (3) \\ (4) \\ (4) \\ (5) $	2)		
	I guaratees all the m	umbers	
	tor x in (a,b)	₩+~~	
a b a b	401 2 31 (0)07		
M between P(a) & \$(b) (3)			
And all the values of	are		
obtained in a x.			
ta) which correspond to			
ta) which correspond to			
a b which correspond to t	nn y vakues		al Parts
$\frac{\frac{1}{2}}{a} \qquad \qquad$	nn y vakes 205y IVT since en is contin words	dpoint given so just ch & f(a) and f(b)	ieck function & the number t
$\frac{\frac{1}{2}(a)}{a}$ which correspond to t a b Application Show there is an x between $[0, \frac{1}{2}]$ such that $\notin e^{\frac{1}{2}}$ $e^{\frac{1}{2}}sin(x) = e^{\frac{1}{2}}$	nn y values 2004 IVT since en is contin wars wort is inbetwer	$en$ $\mp(a) + \pm(b)$ .	leck function & the number t
$\frac{\frac{2}{4}(c_{0})}{a} \qquad \qquad$	nn y values 2004 IVT since en is contin wars wort is inbetwer	Jpaint giv <u>on so just C</u> & ≠Ca) and ≠Ch) en ≠Ca) 4 ≠Ch). <b>functions (e<sup>x</sup> &amp; sin(x))</b>	leck function & the number t
$\frac{\frac{2}{4}(c_{0})}{a} \qquad \qquad$	nn y values 2004 IVT since en is contin wars wort is inbetwer	functions (e <sup>x</sup> & sin(x))	
$\frac{\frac{1}{2}(a)}{a}$ $\frac{\frac{1}{2}(a)}{b}$ $\frac{Application}{Application}$ Show there is an x between $[0, \frac{\pi}{2}]$ such that $(e^{\frac{\pi}{2}})$ $\frac{e^{\frac{\pi}{2}}sin(x) = e^{\frac{\pi}{2}/4}}{b}$ Note $e^{\frac{\pi}{2}}sin(x)$ is continuous on $[0, \frac{\pi}{2}]$ since it is the prime And $e^{\frac{\pi}{2}}sin(\frac{\pi}{2}) = e^{\frac{\pi}{2}/2}$	m y values Easy IVT since env is contin wars want is inbetwer aduct of 2 continuous	functions (e <sup>x</sup> & sin(x))	eck function a the number t $a^{TC}sin(c) = e^{TC}$
$\frac{\frac{2}{4}(c_{0})}{a} \qquad \qquad$	m y values Easy IVT since env is contin wars want is inbetwer aduct of 2 continuous	functions (e <sup>x</sup> & sin(x))	
$\frac{\frac{1}{2}(a)}{a}$ $\frac{\frac{1}{2}(a)}{b}$ $\frac{Application}{Application}$ Show there is an x between $[0, \frac{\pi}{2}]$ such that $(e^{\frac{\pi}{2}})$ $\frac{e^{\frac{\pi}{2}}sin(x) = e^{\frac{\pi}{2}/4}}{b}$ Note $e^{\frac{\pi}{2}}sin(x)$ is continuous on $[0, \frac{\pi}{2}]$ since it is the prime And $e^{\frac{\pi}{2}}sin(\frac{\pi}{2}) = e^{\frac{\pi}{2}/2}$	m y values Easy IVT since env is contin wars want is inbetwer aduct of 2 continuous	functions (e <sup>x</sup> & sin(x))	
$\frac{\frac{1}{2}(c_{1})}{a} \qquad \text{which correspond to t}$ $\frac{\frac{1}{2}(c_{1})}{a} \qquad \text{which correspond to t}$ $\frac{Application}{Application} \qquad \text{Show there is an x between EO, } \frac{\pi}{2}  \text{such that } \leftarrow e^{\frac{\pi}{2}}  such th$	mm y values Easy IVT since env is contin worts wont is inbetween aduct of 2 continuous ere is a c such t	functions ( $e^{x}$ & sin(x)) hat $0 < c < \pi_{j2}$ and e	TC Sin(c) = e
$\frac{\frac{1}{2}(a)}{a}$ $\frac{\frac{1}{2}(a)}{b}$ $\frac{Application}{Application}$ Show there is an x between $[0, \frac{\pi}{2}]$ such that $(e^{\frac{\pi}{2}})$ $\frac{e^{\frac{\pi}{2}}sin(x) = e^{\frac{\pi}{2}/4}}{b}$ Note $e^{\frac{\pi}{2}}sin(x)$ is continuous on $[0, \frac{\pi}{2}]$ since it is the prime And $e^{\frac{\pi}{2}}sin(\frac{\pi}{2}) = e^{\frac{\pi}{2}/2}$	mm y values Easy IVT since env is contin worts wont is inbetween aduct of 2 continuous ere is a c such t	functions ( $e^{x}$ & sin(x)) hat $0 < c < \pi_{j2}$ and e	
$\frac{\mathcal{F}^{(c)}}{a} \qquad \qquad$	mm y values Easy IVT since env is contin worts wont is inbetween aduct of 2 continuous ere is a c such t	functions ( $e^{x}$ & sin(x)) hat $0 < c < \pi_{j2}$ and e	TC Sin(c) = e
$\frac{f(a)}{a}$ which correspond to the correspond	mm y values Easy IVT since env is contin worts wont is inbetween aduct of 2 continuous ere is a c such t	functions ( $e^{x}$ & sin(x)) hat $0 < c < \pi_{j2}$ and e	TC Sin(c) = e
$\frac{(a)}{a}$ $\frac{(a)}{b}$ $\frac{Application}{Application}$ Show there is an x between EO, $\frac{\pi}{2}$ ] such that $(a)$ $\frac{e^{X}sin(X) = e^{T/4}}{e^{X}sin(X) = e^{T/2}}$ Note $e^{X}sin(X)$ is continuous on EO, $\frac{\pi}{2}$ ] since it is the prior And $e^{O}sin(O) = 0$ , $e^{T/2}Sin(T^{T/2}) = e^{T/2}$ . And $O < e^{T/4} < e^{T/2}$ so the Intermediate Value Thrm tells us the Intermediate	mm y values Easy IVT since env is contin worts wont is inbetween aduct of 2 continuous ere is a c such t	functions ( $e^{X}$ & sin(X)) hat $0 < c < \pi/2$ and e	TC Sin(c) = e
$\frac{f(a)}{a}$ which correspond to the correspond	mm y values Easy IVT since env is contin worts wont is inbetween aduct of 2 continuous ere is a c such t	functions ( $e^{X}$ & sin(X)) hat $0 < c < \pi/2$ and e	TC Sin(c) = e
$\frac{f(a)}{a}$ which correspond to the correspond	mm y values Easy IVT since env is contin worts wont is inbetween aduct of 2 continuous ere is a c such t	functions ( $e^{X}$ & sin(X)) hat $0 < c < \pi/2$ and e	TC Sin(c) = e

	Idea: Remnte
Harder Examples)	$e^{c} = \hat{c}^{2} $ as $e^{c} - \hat{c}^{2} = 0$
Example 2 Show that there is an a such that	so it becomes find a zero
$c = \frac{1}{2}$	$G_{\pi} = F(x) = e_{x} - \chi_{\pi}^{2}$
$e^{c} = c^{2}$ $\leftarrow$ harder since you need to find an interval Solution 1) Note $e^{c} = c^{2}$ means $e^{c} - c^{2} = 0$	Finding an x such that f(x)=c Suggests IVT (c=0 in this case
2) Write $f(x) = e^{-x^2}$ so problem wants us to find a c such that $f(c) = 0$	So our goal is to find a <b< td=""></b<>
3) Now as \$(x) is continuous everywhere, so we want to \$ind	f(a) < 0 and $f(b) > 0$ OB
a <b (#ca)="" with="">0 and #cb)&lt;0 [Want #ca) &amp; #cb) to have different signs to</b>	$\begin{array}{c} f_{1}(a) < 0  \text{and}  f_{1}(b) > 0  OR \\ f_{1}(a) > 0  \text{and}  f_{1}(b) < 0  \hline \\ f_{2}(a) > 0  \text{and}  f_{2}(b) < 0  \hline \\ f_{2}(a) < 0  f_{2}(b)  f_{2}(b) < 0  \hline \\ f_{2}(a) < 0  f_{2}(b)  f_{2}(b) < 0  \hline \\ f_{2}(a) < 0  f_{2}(b)  f_{2}(b) < 0  \hline \\ f_{2}(a) < 0  f_{2}(b)  f_{2}(b) < 0  \hline \\ f_{2}(a) < 0  f_{2}(b)  f_{2}(b) < 0  \hline \\ f_{2}(a) < 0  f_{2}(b)  f_{2}(b) < 0  \hline \\ f_{2}(a) < 0  f_{2}(b)  f_{2}(b) < 0  \hline \\ f_{2}(a) < 0  f_{2}(b)  f_{2}(b) < 0  \hline \\ f_{2}(a) < 0  f_{2}(b)  f_{2}(b) < 0  \hline \\ f_{2}(a) < 0  f_{2}(b)  f_{2}(b) < 0  \hline \\ f_{2}(a) < 0  f_{2}(b) < 0  \hline \\ f_{2}(a) < 0  f_{2}(b) < 0  \hline \\ f_{2}(a) < 0  f_{2}(b) < 0  \hline \\ f_{2}(b) < 0  f_{2}(b) < 0  \hline$
100 to use 100 and \$(0) >0 act 0 is between \$(a) & \$(b) to use IVT	Le. f(a) & f(b) have diff Signs to use IVT to get there is a c wlaccob Such that
$\frac{1}{1} \frac{1}{1} \frac{1}$	
$\frac{130-00}{2}$ [2 - 0] [2 - 0] $\pm (0) = e^{0} - 0^{-1} = 1 > 0$	
$f(1) = e - 1 > 0 \leftrightarrow so f(a), f(b) > 0 so Cannot use exercise 2$	
so try new interval (use intuition that $\lim_{x \to \infty} e^{x} = 0$ )	
<u>2nd try</u> a = - 100 b=0	
$\frac{2}{4}(a) = e^{100} - (-100)^2 = e^{-100} - (-100)^2 < 0$	
$f(x) = e^{10^{\circ}} - (-100)^{\circ} = e^{-10^{\circ}} - (-100)^{\circ} < O$ $f(x) = (-70)^{\circ} - (-70)^{\circ} < O$	
So $f(b) > 0$ and $f(a) < 0$ with $f$ continuous on $E^{-109/0}$	(1)=()
so () is between f(a) & f(b) which Implies there is a c such that -100 <c< ()="" f<="" td="" with=""><td></td></c<>	
$c.e. = e^{c} - c^{2} = 0  c.e.  e^{c} = c^{2}  D$	