Logistics

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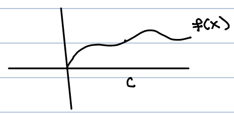
Read by yourself

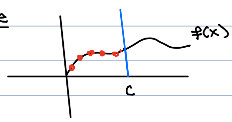
Limit: Given a function $f(x)$ and a number c



We say that the limit as x approaches from below c for f is the value $f(x)$ approaches for $\boxed{x<c}$ as x gets closer & closer to c while x is strictly less than c

denoted as $\lim_{x\to c^-} f(x)$ [also called left limit]

Picture



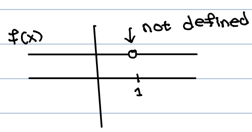
$\lim_{x\to c^-} f(x)$ is what the red dots approach when it gets closer & closer to the blue line

Note the red dots x coordinate are strictly less than c

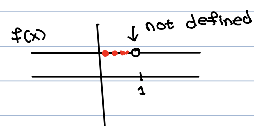
This is important since f may not be defined at c but its limit can be!

For example $f(x) = \frac{x-1}{x-1}$ is not defined at $x=1$ (since plugging in 0 gives 0/0 which is indeterminate)

but for any $x \neq 1$ we have $f(x) = 1$



but



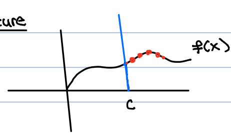
but • approaches 1

so $\lim_{x\to 1^-} f(x) = 1$ but $f(1)$ is indeterminate

The right limit at c for $f(x)$ is the value $f(x)$ approaches for $\boxed{x>c}$ as x gets closer & closer to c while x is strictly greater than c

This is written $\lim_{x\to c^+} f(x)$

Picture



$\lim_{x\to c^+} f(x)$ is what the red dots approach when it gets closer & closer to the blue line

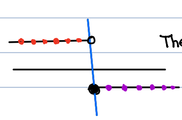
We say the limit of f as x approaches c is L if

$\lim_{x\to c^+} f(x) = \lim_{x\to c^-} f(x) = L$

and write $\lim_{x\to c} f(x) = L$

Examples (Left limit may not equal right limit and may not equal $f(c)$)

$$f(x) := \begin{cases} 1 & \text{if } x<0 \\ -1 & \text{if } x \geq 0 \end{cases}$$



The red dots approach 1 as they approach the blue line $(x=0)$: i.e. $\lim_{x\to 0^-} f(x) = 1$

The purple dots approach -1 as they approach the blue line $(x=0)$: i.e. $\lim_{x\to 0^+} f(x) = -1$

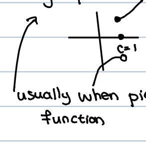
And $f(0) = 1$ but $\lim_{x\to 0^+} f(x) = -1 \neq 1 = \lim_{x\to 0^-} f(x)$ & $\lim_{x\to 0^-} f(x) \neq f(0)$

[Section 2.5 of Textbook]

Evaluating Limits Given a function $f(x)$ and a number c. Find $\lim_{x\to c} f(x)$ if it exists. solution procedure (in general)

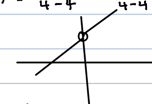
May not be comprehensive, but can solve most problems

① First check if f is continuous at $x=c$ [i.e. no jump discontinuity, indeterminate, zero on denominator

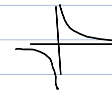


usually when piecewise function

$f(x)=\frac{x^2-16}{x-4}$ " $f(4)=\frac{(4)^2-16}{4-4}=\frac{16-16}{4-4}=\frac{0}{0}$ "



$f(x)=\frac{1}{x}$ " $f(0)=\frac{1}{0}$ "



or $\frac{\infty}{\infty}$, $\frac{-\infty}{\infty}$]

② If f is continuous then $\lim_{x\to c} f(x)=f(c)$

i.e. $\lim_{x\to 2}\frac{1}{x}=\frac{1}{2}$, $\lim_{x\to 5}\frac{x^2-16}{x-4}=\frac{(5)^2-16}{5-4}=\frac{25-16}{1}=9$

but $\frac{1}{x}$ and $\frac{x^2-16}{x-4}$ are discontinuous at $x=0$ & $x=4$

③ If $f(x)$ is a rational function i.e. $f(x)=\frac{p(x)}{q(x)}$ [p & q are polynomials] such that "$f(c)=\frac{0}{0}$" factor p & q!

Example $f(x)=\frac{x^2-25}{x-5}$ is indeterminate at $x=5$

then notice $(x^2-25)=(x-5)(x+5)$

since $(x-5)(x+5)=x^2+5x-5x-25=x^2-25$

So for any $x\neq 5$ we have $f(x)=\frac{(x-5)(x+5)}{x-5}=\frac{\cancel{(x-5)}(x+5)}{\cancel{x-5}}=x+5$

Remember for limits we see what happens for $x\neq 5$ what $f(x)$ approaches as x gets closer and closer to 5.

so $\lim_{x\to 5} f(x)=\lim_{x\to 5}(x+5)$ [we approach but don't touch 5] use (2)

to get $\lim_{x\to 5}(x+5)=10$.

④ If $f(x)$ is of the form $\frac{\sqrt{x}-a}{g(x)}$ or $\frac{g(x)}{\sqrt{x}-a}$ and at $x=c$ this is indeterminate (this implies $\sqrt{c}-a=0 \Rightarrow c=\sqrt{a}$)

Try multiplying by the conjugate : $\left(\frac{\sqrt{x}+a}{\sqrt{x}+a}\right.$ ← conjugate)

Example) $f(x)=\frac{\sqrt{x}-3}{x-9}$ is indeterminate at $x=9$

The conjugate is $\frac{\sqrt{x}+3}{\sqrt{x}+3}=1$ for any x

So $f(x)=f(x)\cdot 1=\left(\frac{\sqrt{x}-3}{x-9}\right)\left(\frac{\sqrt{x}+3}{\sqrt{x}+3}\right)=\frac{x-9}{(x-9)(\sqrt{x}+3)}$ ← see week 0 notes if you're confused

$=\frac{1}{\sqrt{x}+3}$ for any $x\neq 9$

so $\lim_{x\to 9} f(x)=\lim_{x\to 9}\frac{1}{3+\sqrt{x}}=\frac{1}{3+\sqrt{9}}=\frac{1}{6}$ by ②

⑤ If it's an indeterminate trig function divided by another i.e. $\frac{\tan(x)}{\sec(x)}$ at $\pi/2$ ($\frac{\tan(\pi/2)}{\sec(\pi/2)}=\frac{\infty}{\infty}$)

try trig identites i.e.

$\tan(x)=\frac{\sin(x)}{\cos(x)}$ ⟶ $\frac{\tan(x)}{\sec(x)}=\frac{\sin(x)}{\cos(x)}\left(\frac{\cos(x)}{1}\right)=\sin(x)$ [when $\cos(x)\neq 0$]

$\sec(x)=\frac{1}{\cos(x)}$

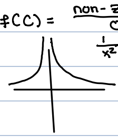
For x close but not $\pi/2$ $\cos(x)\neq 0$

so $\lim_{x\to\pi/2}\tan(x)/\sec(x)=\lim_{x\to\pi/2}\sin(x)=1$

Sometimes the limit does not exist (DNE)

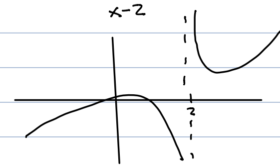
If "$f(c)=\frac{\text{non-zero}}{0}$" the limit does not exist

$\frac{1}{x^2}$



$f(x)=\frac{1}{x^2}$ has no limit at zero since "it approaches ∞" infinite discontinuity

$\frac{x^2}{x-2}$ infinite discontinuity



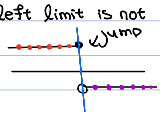
– Recall

We say the limit of f as x approaches c is L if $\lim_{x\to c^+} f(x)=\lim_{x\to c^-} f(x)=L$

and write $\lim_{x\to c} f(x)=L$

So if the left limit is not equal to right limit then the limit DNE

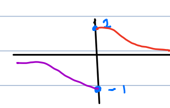
Ex)



↙jump

left limit ≠ right limit

↑usually when piece-wise function



↙jump discontinuity

left limit ≠ right limit

Common Denominator

If we have an expression of the form

$$\lim_{x\to c} \frac{f(x)}{g(x)} - \frac{h(x)}{u(x)}$$

$$\& \quad \frac{f(c)}{g(c)} - \frac{h(c)}{u(c)} = \text{"} \frac{\text{non-zero number}}{0} - \frac{\text{non-zero number}}{0} \text{"}$$

Then make them have the same denominator

Example $$\lim_{x\to 0}\left[\frac{3}{x} - \frac{3}{x^2+x}\right] = \lim_{x\to 0}\left[\frac{3}{x} - \frac{3}{x(x+1)}\right] = \lim_{x\to 0}\left[\frac{3(x+1)}{x(x+1)} - \frac{3}{x(x+1)}\right] = \lim_{x\to 0}\left[\frac{3x}{x(x+1)}\right] = \lim_{x\to 0}\left[\frac{3}{x+1}\right] = 3$$

Warning some limits do not exist! Algebra tricks help us see if the indeterminate has a limit or not

$$\lim_{x\to 0}\left[\frac{1}{x} - \frac{1}{x^2}\right] = \lim_{x\to 0}\left[\frac{x-1}{x^2}\right] \quad \text{"} \frac{-1}{0} \text{"}$$ infinite discontinuity