Then the permutation
\[
\begin{pmatrix}
1 & 2 & 3 \\
3 & 1 & 2
\end{pmatrix}
\]
will reshuffle the figures into the following order.

Problem 5 For the original order of figures given on page 5, write down the permutations that correspond to the following pictures.

\[
\begin{pmatrix}
1 & 2 & 3 \\
1 & 3 & 2
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 2 & 3 \\
2 & 3 & 1
\end{pmatrix}
\]
Note that the last permutation does not reshuffle anything at all. Permutations of this kind typically denoted as \( e \) and called trivial. A trivial permutation is still a permutation, and an important one!

**Problem 6** Write down the trivial permutation for \( n = 5 \).

\[
\begin{pmatrix}
1 & 2 & 3 & 4 & 5 \\
1 & 2 & 3 & 4 & 5
\end{pmatrix}
\]

**Problem 7** For the original order of figures given on page 5, draw the figures in the orders prescribed by the permutations below. Use the space to the right of a permutation to draw the corresponding picture.

\[
\begin{pmatrix}
1 & 2 & 3 \\
3 & 2 & 1
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 2 & 3 \\
2 & 1 & 3
\end{pmatrix}
\]
Recall that \( n! = 1 \times 2 \times \ldots \times (n - 1) \times n \). For example, 
\( 3! = 1 \times 2 \times 3 = 6 \).

**Problem 8** Compute \( 5! \)

\[
5! = 5 \times 4 \times 3 \times 2 \times 1 = 120
\]

**Problem 9** How many permutation of four elements are there?

For the first place, we have 4 choices. 

\[
\begin{array}{cccc}
\text{2nd} & \text{3rd} & \text{4th} & \text{1st} \\
\text{----} & \text{----} & \text{----} & \text{----}\\
\end{array}
\]

Thus, we have \( 4 \times 3 \times 2 \times 1 = 4! = 24 \) permutations.

**Problem 10** How many permutation of \( n+1 \) elements are there?

Similarly to what we did above, for the 1st place, \( (n+1) \) choices of elements.

In total, there will be \( (n+1)! \) permutations.

**Problem 11** Write down a permutation of four elements.

\[
\begin{pmatrix}
1 & 2 & 3 & 4 \\
1 & 2 & 3 & 4
\end{pmatrix}
\]

Note: There are many permutations (24, to be precise).

**Problem 12** Write down a permutation of four elements that keeps the third element in place.

\[
\begin{pmatrix}
1 & 2 & 3 & 4 \\
2 & 3 & 4 & \text{X}
\end{pmatrix}
\]

Note that there will be \( 3 \times 2 \times 1 \times 1 = 6 \) permutations satisfying this standard. (The 1st place has 3 choices of element, 2nd has 2, and 3rd has 1.)
Find the number of permutations of four elements that keep the third element in place.

The number $= 3 \times 2 \times 1 \times 1 = 6$

It is possible to combine, or *multiply*, permutations. For example, let us apply the permutation

$$\delta = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$

to the marbles already reshuffled by the permutation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}.$$  

The permutation $\delta$ switches the first and second elements, so

$$\delta \circ \sigma = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}.$$ 

Let us take another look at the above computation using the figures from page 5. Originally, the set of the figures is ordered as follows.

\[ \square \quad \triangle \quad \bigcirc \]
The permutation

\[ \sigma = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \]

produces the picture below.

The permutation

\[ \delta = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \]

applied to the latter configuration gives us the following.

Comparing the last picture to the original gives us the answer.

\[ \delta \circ \sigma = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \]

Note that in the product \( \delta \circ \sigma \) of permutations, it is the one on the right, \( \sigma \), that acts first on the set it permutes!