GEOMETRY - FINDING ANGLES

MATH CIRCLE (INTERMEDIATE) 02/24/2013

(1) (Introduction) Here are some basic facts that will help us calculate the angles of geometric figures...
   (a) The sum of the angles in any triangle is...
   (b) A pair of vertical angles are...
   (c) Angles lying along a straight line add up to...
   (d) An inscribed angle equals half the central angle which intercepts the same ________ of a circle.
   (e) Two inscribed angles intercepting the same arc of a circle are...
   (f) Rigid motions of the plane do not change...

(2) Angle bisector $BK$ is drawn in isosceles triangle $ABC$, with $AB = AC$ and angle $A$ equal to 36 degrees. Prove that $BK = BC$. 
(3) Find the sum of the angles at the vertices of a five-pointed star.

(4) Can two angle bisectors in a triangle be perpendicular? Justify your answer.

(5) Chords $AB$ and $CD$ in circle $S$ are parallel. Prove that $AC = BD$. 
(6) The ratio of three consecutive angles in an inscribed quadrilateral is $2 : 3 : 4$. Find the measure of each angle.

(7) In triangle $ABC$, the measure of angle $A$ is 90 degrees. Median $AM$, angle bisector $AK$, and altitude $AH$ are drawn. Prove that angles $MAK$ and $KAH$ are congruent. (Note: A median connects a vertex with the midpoint of the opposite side of the triangle. An altitude connects a vertex with the opposite side of the triangle, intersecting it at a 90-degree angle.)

(8) Square $ABCD$ is given. A circle with radius $AB$ and center $A$ is drawn. This circle intersects the perpendicular bisector of $BC$ in two points, of which $O$ is the closest to $C$. Find the measure of angle $AOC$. 
(9) Two circles intersect at points $A$ and $B$. $AC$ is a diameter of the first circle, and $AD$ is a diameter of the second. Prove that points $B$, $C$, and $D$ lie on the same straight line.

- Math Kangaroo Problems -

(10) In the picture, the points $Q$, $S$, and $R$ are collinear, the measure of angle $QPS$ is 12 degrees, and $|PQ| = |PS| = |RS|$. What is the measure of angle $QPR$?

\[\text{Figure 0.1. Problem 10.}\]
(11) Quadrilateral $ABCD$ is a square (see the picture). The measure of $\angle OND$ is $60^\circ$. What is the measure of $\angle COM$?

Figure 0.2. Problem 11.

(12) What fraction of the largest square is the shaded region, by area?

Figure 0.3. Problem 12.
(13) The figure in the picture is a regular nonagon (a regular 9-sided polygon). What is the measure of angle $\alpha$?

\begin{figure}[h]
\centering
\includegraphics[width=0.3\textwidth]{figure04.png}
\caption{Problem 13.}
\end{figure}

(14) Triangles $ABC$ and $CDE$ are equilateral and congruent. If the measure of $\angle AC\!D = 80^\circ$, what is the measure of $\angle AB\!D$?

\begin{figure}[h]
\centering
\includegraphics[width=0.3\textwidth]{figure05.png}
\caption{Problem 14.}
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