Number line and Coordinates on the Plane

Part I

The Number Line

Take a look at the number line below.

![Number Line Diagram]

Position of a point on the number line is determined by just one number. A number line should have the following features:

- A *starting point* (point marked 0);
- *Scale*: we need to know how long the unit length is; One way to indicate the scale is to mark where the number 1 is;
- *Direction*: we need to know the direction in which the numbers are increasing. The direction of increase is marked by an arrow.

Once a starting point and a scale are chosen, every point on the number line corresponds to a number. The usual agreement is that positive numbers are pictured to the right of 0 while negative numbers are pictured to the left of 0. Thus, the numbers increase as we move from left to right. This is indicated by an arrow at the right end of the number line.
1. On the number line below mark the points 2, 4, −1, −3.

2. Gina marked the numbers 2 and 3 on the number line and then erased 0 and 1. Can you find where 0 and 1 on the number line are? Can you mark the direction of the number line with an arrow?

3. Carl marked the numbers 2, 3 and 5 on the number line, but erased 0 and 1. The teacher tells him that he did not mark the points correctly. How does she know?
4. Mark the numbers 2, 3, 4, 5 and \(-1, -2, -3, -4, -5\) on the number below. Then answer the questions under the picture:

(a) What is the distance between the points \(-2\) and 3 on the number line?

(b) What is the distance between the points \(-5\) and \(-1\) on the number line?
Part II

Coordinates on a plane

If we want to indicate the position of a point on the plane, one number is no longer enough. Suppose we mark a point on the plane and call it point \( O \).

1. How can you describe the position of a point \( A \) relative to the point \( O \)? Take a look at the picture below and explain where \( A \) is relative to \( O \).

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\text{If your description is in words, can you use just one number to specify where } A \text{ is in relation to } O? 
\]
2. A coordinate system on the plane is a way to describe position of any point on the plane relative to a fixed point (the center of coordinate system). A coordinate system has

(a) Center (point O);
(b) Two coordinate axes (the $x$-axis and the $y$-axis);
(c) A scale (unit length) chosen on every axis.

Use the coordinate system below to answer the following questions:

- Is it true that each coordinate axis is a number line?

- Mark point $A$ on the coordinate system so that:
  - Point $A$ lies on the $x$-axis;
- Point \( A \) is at the mark 5 on the \( x \)-axis;

- Mark point \( B \) on the coordinate system so that:
  - Point \( B \) lies on the \( y \)-axis;
  - Point \( B \) is at the mark 2 on the \( y \)-axis;

3. There is an organized way to write coordinate point locations. For consistency, we read the \( x \)-value first and then the \( y \)-value. This is exactly how a coordinate point is written: \((x, y)\).

Put the coordinate axes onto the geoboard you were given in class. Use a red peg to indicate the intersection of the coordinate axes. This is the point \( O \) with coordinates \((0, 0)\). Use the red pegs to mark the points with the following coordinates:

- \((2, 2)\)
- \((2, 0)\)
- \((1, 1)\)
- \((2, -2)\)
- \((-2, -2)\)
- \((-1, 1)\)
- \((-2, 0)\)
- \((-2, 2)\)
- \((0, 0)\) (you already have a red peg there!)

The pegs now form a shape resembling a letter. Which letter is it?
Now, put all the blue pegs back into the bag.

4. Imagine that you put a coordinate plane over a map. Emmanuelle starts walking from $O$ (the center of coordinate system) following the directions below:

(a) First, she walks 3 units north;
(b) Then, she goes 3 units east;
(c) After that, she goes 1 unit south;
(d) Finally, she goes 2 units west;

What are the coordinates of a point where Emmanuelle ends up?

5. Cory also starts walking from the point where Emmanuelle stopped following the directions below:

(a) First, he walks 1 unit south;
(b) Then, he walks 3 units west;
(c) After that, he goes 5 unit north;
(d) Finally, he goes 2 units east;

What are the coordinates of a point where Cory ends up?
6. Use the Geoboard to find the distances between the following pairs of points:
   (a) (0, 0) and (2, 0); _____
   (b) (5, 0) and (0, 0); _____
   (c) (2, 3) and (7, 3); _____
   (d) (5, 1) and (5, 4); _____

7. Let’s take a look at the points with coordinates (0, 0) and (3, 3).
   (a) Alex says that the distance between these points is 3. Do you think he is right? If not, do you think the distance is actually bigger or smaller than 3? Why?
   (b) Beth says that the distance between these points is 6. Do you think she is right? If not, do you think the distance is actually bigger or smaller than 6? Why?
8. Plot the points (4, 4) and (4, 8). Using your rubber bands and pegs, find all the possible squares that can be made, using these two points and 2 of the 4 vertices. (Hint: There are more than 2 possibilities.)

Graph each answer on a plane below. You may or may not need all of them.
9. Points $A$, $B$, $C$ and $D$ are on the same line so that:
   - The distance between $A$ and $C$ is 2 cm;
   - The distance between $B$ and $D$ is 5 cm;
   - The distance between $D$ and $C$ is 15 cm;

What is the distance between $B$ and $C$?

Draw a picture of your solution:

10. 8 girls went on a field trip. Of them,
   - 3 girls each had 2 sisters who went on the trip;
   - 4 girls each had 1 sister who went on the trip;
   - 1 girl did not have a sister on the field trip.

   The mothers of all those girls went with them. How many mothers were there?