Projections and Levels

October 12, 2012

Today we will use levels to describe solids built over the base of size $2 \times 2$ and $3 \times 3$. If you think of your solid as a house, each level represents a different floor. For a solid over base of size $3 \times 3$, there are 3 levels. We will draw how each level looks like (from the top). This information is enough to build the solid and study its properties.

Throughout the handout, L1 is the lowest (bottom) level, L2 is the middle level, and L3 is the top level.

1. Build the solid that has the following levels:

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   \end{center}

   \begin{center}
   \text{L1} \quad \text{L2} \quad \text{L3}
   \end{center}

   Show your solid to the instructor at your table.
2. Use your blocks to do the following:

(a) Build two solids over the base of size $3 \times 3$ that are different but such that all three projections are the same.

(b) Picture the levels of these solids by drawing around the blocks (as in the problem above):

**Solid 1**

![Solid 1 Diagram]

(c) Are there two different solids such that all three of their projections are the same?
3. Use your blocks to do the following:

(a) Build the solid that has the following levels:

(b) Imagine that level 1 is built out of red cubes, level 2 is built out of blue cubes, and level 3 is built out of green cubes. Color the levels above accordingly:

- Level 1: color red (6 boxes);
- Level 2: color blue (4 boxes);
- Level 3: color green (1 box);

(a) For each of the projections below, do the following:

- If there is a cube in the top level (L3) that shows up in the projection, color it green;
- If there is a cube in the middle level (L2) that shows up in the projection, color it blue;
- If there is a cube in the bottom level (L1) that shows up in the projections, color it red;
4. Use your blocks to do the following:

(a) Build the solid that has the following levels:

(b) Color the levels:
- Level 1: color red (8 boxes);
- Level 2: color blue (7 boxes);
- Level 3: color green (2 boxes).

(a) For each projection below, do the following:
- If there is a cube in L3 that shows up in the projection, color it green;
- If there is a cube in L2 that shows up in the projection, color it blue;
- If there is a cube in L1 that shows up in the projection, color it red.
5. Use your blocks to do the following:

(a) Build the solid that has the following levels:

(b) As before, color the levels (L1 – red; L2 – blue; L3 – green);

(c) For each projection below, do the following:

- If there is a cube in L3 that shows up in one of the projections, color it green.
- If there is a cube in L2 that shows up in one of the projections, color it blue.
- If there is a cube in L1 that shows up in one of the projections, color it red.
6. Explain why the following solids cannot be built out of cubes. Indicate the problem areas by shading them on the picture:

(a)

(b)

(c)
7. Given that:
   - L1 has 5 squares;
   - L2 has 3 squares;
   - L3 has 2 squares;

   Decide how many blocks you need to build the solid.

8. More generally, how can you get the number of blocks you need to build the solid from the pictures of the levels?

9. Now let’s try the opposite way. Given the three projections, draw the levels of the solid with these projections.
10. CHALLENGE! Draw levels of the solids with the given projections:

(a) 

![TOP](image1) ![FRONT](image2) ![SIDE](image3)

(b) 

![TOP](image4) ![FRONT](image5) ![SIDE](image6)

L1 L2 L3
11. We saw that given three projections, sometimes we can build several different solids with these projections. Now imagine that the squares in the projections are colored by red, blue and green according to whether the corresponding cube is in level 1, 2 or 3. Do you think such a colored projection defines the solid? Explain.

12. Build your own solid and write your own question about it. Exchange the problems with your partner and solve each other’s problems.
1. Last week, we learned what a Caesar Cipher was. Your homework was to cut one out and bring it to class to use. Let’s practice with some shifting first, with your ciphers:

Fill in the key for a Caesar cipher with a shift of +4:

<table>
<thead>
<tr>
<th>Letter:</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
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<tbody>
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Now, decode the following message:

**CSY EVI KIXXMRK KSSH EX XLMW!**

2. Now that you have the hang of it, think of a shift for your own message. Write your message below and use the white piece of paper given to you by your instructor to encode your message. Then hand it to a partner and see if they can figure out your message!

Your message: